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REJECTION OF UNKNOWN SINUSOIDAL DISTURBANCE FOR LINEAR SYSTEMS WITHOUT ZEROS

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Abstract

This paper proposes an output feedback controller for linear systems which contain sinusoidal exogenous inputs with unknown frequency magnitude, and phase. Unlike previous studies, we do not assume that the order of the exosystem is known. The global asymptotic convergence of the output error to zero is guaranteed under the assumption that an upper bound is known on the order of the exosystem and the plant does not have zeros, even though the proposed method does not identify the actual frequencies of the unknown exogenous input.

Introduction

The problem of a systems asymptotically tracking prescribed reference inputs and/or asymptotically rejecting undesired disturbances is critical and significant in control theory. Solution to this problem has been actively studied since 1970 and coined as *output regulation* problem in the literature (for example, [1]-[3]). Based on the theory, we consider the problem of as linear system with the *exogenous input* representing the reference inputs and/or the disturbances, which is sinusoidal signals with unknown frequency, magnitude, and phase. In the literature of output regulation, the exogenous input is generated by an *exosystem* [3].

For the known exogenous input, the solution to this problem is a very natural one [1], [3], [4]. On other hand, for the unknown exogenous input, some related works for linear systems can be found in [5]-[7]. The method provided in [5] guarantees exponential tracking of the reference and/or rejecting of the disturbance. However, the method requires the order of the exosystem. To overcome this constraint, in [6], it is shown that the regulation problem is solvable under the assumptions that an upper bound on the order of the exosystem is known and the plant is linear minimum phase systems. For non-minimum phase linear systems, a solution with an indirect adaptive approach has be proposed [7]. However, as shown in [6], the drawbacks of this approach is not popular in practive because of its complexity and some singularity problems for the computation of the controller.

The main contribution is to propose a numerically efficient approach to the design of global asymptotically stable output feedback controller for linear systems without zeros. The approach is designed by the help of adaptive observer developed in [8]. In particular, it requires only the upper bound on the order of the exosystem similar to the assumption in [6], [7]. However, the algorithm presented in this paper is provided that is simpler than that in [6] and does not involve any singularity problem of [7] because of the assumption that the plant does not have zeros. The proposed controller achieve the global asymptotic convergence of the output error to zero, even if it does not recover the actual frequencies since the *persistence of excitation (PE)* does not satisfied by the uncertain order of the exosystm [9].

Problem Statement

The regulation problem is formulated for linear time-invariant (LTI) single-input-single-output (SISO) systems modeled as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bu(t) + Pw(t),\\ e(t) &= cx(t) + qw(t),\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, w is the exogenous input which includes reference (to be tracked) and/or disturbance (to be rejected), and $e \in \mathbb{R}$ is the output to be regulated to zero. The matrices A, b, and c are known, but P and q are unknown. It is assumed that e can be measured while x and w are not measurable. We suppose that the pair (A, b) is controllable and (A, c) is observable. In addition, we assume that, for some positive m, the exogenous input w is generated by an exosystem

$$\dot{w}(t) = Sw(t), \tag{2}$$

where

$$S = \mathrm{diag}(\sigma_1 S_o, \sigma_2 S_o, \cdots, \sigma_m S_o), \quad S_o = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

in which $\sigma_1, \sigma_2, \cdots, \sigma_m$ are unknown distinct positive constants. Since the exosystem is only driven by the initial condition $w(0)$ and there is no assumption for the initial condition, some mode of the exosystem may be zero. Thus, the twice that of the positive integer m actually represents an upper bound on the order of the exosystem.

The problem considered in this paper can be stated as follows. Given system (1) and exosystem (2), find a dynamic error feedback controller of the form

$$\begin{aligned} \dot{z}(t) &= f(t,z,u,e), \\ u(t) &= h(t,z), \end{aligned} \tag{3}$$

such that $\lim_{t\to\infty} e(t) = 0$ and all the states of the closed-loop system are bounded. To solve the problem, we pose some conditions on which the proposed controller is based.

Assumption 1: The positive integer m is known, while some parts of the exosystem's initial condition can be zero.◇

In order to regulate e to zero, we assume the following.

Assumption 2: The plant (1) does not have zeros. ◇

By virtue of Assumption 2, it can be assumed that A, b, and c are given by

$$A = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T,$$

where $b_1 = b_2 = \cdots = b_{n-1} = 0$ and $b_n \neq 0$. Also, it follows from [3, Theorem 1.9] that there exist matrices $\Pi \in \mathbb{R}^{n\times 2m}$. and $\gamma \in \mathbb{R}^{1\times 2m}$ such that

$$\begin{aligned} \Pi S &= A\Pi + b\gamma + P, \\ 0 &= c\Pi + q. \end{aligned} \tag{4}$$

Main Result

The following theorem represents the main result given in this paper.

Theorem 1: Consider the systems (1) and (2) under Assumption 1 and 2, there exists a dynamic output feedback controller such that, for any initial condition: i) all the states of the overall closed-loop system are bounded and ii) $\lim_{t\to\infty} e(t) = 0$. ◇

First, we introduce the output feedback controller. Then, a proof of Theorem 1 will be given.

A. Controller Design

To solve the regulation problem, we design the dynamic output feedback controller as follows:

$$\begin{aligned} \dot{\hat{\xi}} &= A_c\hat{\xi} + b_c u + \Psi(e,u)\hat{\theta} + L(e - c_c\hat{\xi}) + \Xi\dot{\hat{\theta}}, \\ \dot{\hat{\theta}} &= K_{ad}\Xi^T c_c^T (e - c_c\hat{\xi}), \\ \dot{\Xi} &= (A_c - Lc_c)\Xi + \Psi(e,u), \\ u &= \begin{bmatrix} K & \bar{\gamma} \end{bmatrix} T_c^{-1}(\hat{\theta})\hat{\xi}, \end{aligned} \tag{5}$$

where

$$A_c = \begin{bmatrix} -a_1 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -a_n & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}, \; b_c = \begin{bmatrix} b_1 \\ \vdots \\ b_n \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{aligned} c_c &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}, \\ \Psi(e,u) &= \begin{bmatrix} -a[1]e & -a[2]e & \cdots & -a[m]e \end{bmatrix} \\ &\quad + \begin{bmatrix} b[1]u & b[2]u & \cdots & b[m]u \end{bmatrix}, \end{aligned}$$

in which, $a[i] \in \mathbb{R}^{n+2m}$ and $b[i] \in \mathbb{R}^{n+2m}$, $i = 1,2,\cdots,m$, are given by

$$\begin{aligned} a[1] &= \begin{bmatrix} 0 & 1 & a_1 & \cdots & a_n & 0 & \cdots & 0 \end{bmatrix}^T, \\ b[1] &= \begin{bmatrix} 0 & 0 & b_1 & \cdots & b_n & 0 & \cdots & 0 \end{bmatrix}^T, \\ a[2] &= \begin{bmatrix} 0 & 0 & 0 & 1 & a_1 & \cdots & a_n & 0 & \cdots & 0 \end{bmatrix}^T, \\ b[2] &= \begin{bmatrix} 0 & 0 & 0 & 0 & b_1 & \cdots & b_n & 0 & \cdots & 0 \end{bmatrix}^T, \\ &\vdots \\ a[m] &= \begin{bmatrix} 0 & \cdots & 0 & 1 & a_1 & \cdots & a_n \end{bmatrix}^T, \\ b[m] &= \begin{bmatrix} 0 & \cdots & 0 & 0 & b_1 & \cdots & b_n \end{bmatrix}^T. \end{aligned}$$

Here, the design parameters K and L are chosen such that $A + bK$ and $A_c - Lc_c$ are Hurwitz, respectively, and $K_{ad} \in \mathbb{R}^{m\times m}$ is any symmetric positive-definite matrix. In addition, $T_c^{-1}(\hat{\theta})$ is made by the following equation θ instead of $\hat{\theta}$,

$$T_c(\theta) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \bar{\alpha}_1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{\alpha}_{n+2m-2} & \bar{\alpha}_{n+2m-3} & \cdots & 1 & 0 \\ \bar{\alpha}_{n+2m-1} & \bar{\alpha}_{n+2m-2} & \cdots & \bar{\alpha}_1 & 1 \end{bmatrix} \begin{bmatrix} \bar{c} \\ \bar{c}\bar{A} \\ \vdots \\ \bar{c}\bar{A}^{n+2m-2} \\ \bar{c}\bar{A}^{n+2m-1} \end{bmatrix}, \tag{6}$$

where $\bar{A}=\begin{bmatrix} A & -b\bar{\gamma} \\ 0 & \bar{S}\end{bmatrix}, \bar{c}=\begin{bmatrix} c & 0\end{bmatrix}$, $\bar{\gamma}=\begin{bmatrix} 1 & 0 & \cdots & 0\end{bmatrix}$,

$$\bar{S}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ -\theta_1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\theta_m & 0 & 0 & \cdots & 0\end{bmatrix}, \theta=\begin{bmatrix}\theta_1 \\ \theta_2 \\ \vdots \\ \theta_{m-1} \\ \theta_m\end{bmatrix},$$

and $\bar{\alpha}_1,\bar{\alpha}_2,\cdots,\bar{\alpha}_{n+2m-1}$ are the coefficients of the characteristic polynomial of $\bar{A}$, i.e.,

$$\begin{aligned}\det(sI-\bar{A}) &= \det(sI-A)\cdot\det(sI-\bar{S}) \\ &= (s^n+a_1s^{n-1}+a_2s^{n-2}+\cdots+a_n) \\ &\quad\cdot(s^{2m}+\theta_1s^{2m-2}+\theta_2s^{2m-4}+\cdots+\theta_m) \\ &= s^{n+2m}+\bar{\alpha}_1s^{n+2m-1}+\cdots+\bar{\alpha}_{n+2m-1}s+\bar{\alpha}_{n+2m}.\end{aligned} \tag{7}$$

Here, $\theta_1,\theta_2,\cdots,\theta_m$ are given by

$$\theta_1=\sum_{i=1}^{m}\sigma_i^2,\theta_2=\sum_{i_1<i_2}\sigma_{i_1}^2\sigma_{i_2}^2,\cdots,\theta_m=\sum_{i_1<\cdots<i_m}\sigma_{i_1}^2\cdots\sigma_{i_m}^2$$

with $i_1,i_2,\cdots,i_m\in\{1,2,\cdots,m\}$.

Remark 1: In the designed controller (5), the adaptive observer parts are designed based on the proposed method in [8]. ◇

B. Stability Analysis

In order to prove Theorem 1, some pre-works are required. Let $x_r:=\Pi w$, $u_r:=\gamma w$, and $\tilde{x}:=x-x_r$. Then, it follows from (4) that

$$\begin{aligned}\dot{\tilde{x}} &= A\tilde{x}-bu_r+bu, \\ e &= c\tilde{x}.\end{aligned} \tag{8}$$

Define, with $\sigma=\begin{bmatrix}\sigma_1 & \sigma_2 & \cdots & \sigma_m\end{bmatrix}^T$,

$$T_e(\sigma)=\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \alpha_1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{2m-2} & \alpha_{2m-3} & \cdots & 1 & 0 \\ \alpha_{2m-1} & \alpha_{2m-2} & \cdots & \alpha_1 & 1\end{bmatrix}\begin{bmatrix}\gamma \\ \gamma S \\ \vdots \\ \gamma S^{2m-2} \\ \gamma S^{2m-1}\end{bmatrix} \tag{9}$$

where $\alpha_1=0$, $\alpha_2=\theta_1$, $\alpha_3=0$, $\alpha_4=\theta_2$, $\cdots$, $\alpha_{2m-1}=0$, $\alpha_{2m}=\theta_m$. Now, with $\bar{w}:=T_e(\sigma)w$, the exosystem (2) and $u_r=\gamma w$ are transformed into an observable canonical form

$$\begin{aligned}\dot{\bar{w}} &= \bar{S}\bar{w}, \\ u_r &= \bar{\gamma}\bar{w}.\end{aligned} \tag{10}$$

From the equations (8) and (10), we rewritten as

$$\begin{aligned}\begin{bmatrix}\dot{\tilde{x}} \\ \dot{\bar{w}}\end{bmatrix} &= \begin{bmatrix} A & -b\bar{\gamma} \\ 0 & \bar{S}\end{bmatrix}\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix}+\begin{bmatrix} b \\ 0\end{bmatrix}u, \\ e &= \begin{bmatrix} c & 0\end{bmatrix}\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix}.\end{aligned} \tag{11}$$

The system (11) is transformed into, with $\xi:=T_c(\theta)\begin{bmatrix}\tilde{x}^T & \bar{w}^T\end{bmatrix}^T$,

$$\begin{aligned}\dot{\xi} &= A_c\xi+b_cu+\Psi(e,u)\theta, \\ e &= c_c\xi.\end{aligned} \tag{12}$$

The following lemma indicates that $T_c(\theta)$ can be used as a state transformation matrix for all $\theta\in\mathbb{R}^m$.

Lemma 1: $T_c(\theta)$ is nonsingular for all $\theta\in\mathbb{R}^m$ if Assumption 2 holds and the pair (A,c) is observable. ◇

Proof: From the definition of $T_c(\theta)$, it suffices to show that $(\bar{A},\bar{c})$ is observable for any q, which is equivalent that the matrix

$$\begin{bmatrix}\begin{bmatrix} A-\lambda I \\ c\end{bmatrix} & \begin{bmatrix} -b & 0 \\ 0 & 0\end{bmatrix} \\ 0_{2m\times n} & \bar{S}-\lambda I\end{bmatrix} \tag{13}$$

has full column rank for each l, which is an eigenvalue of either A or $\bar{S}$. Let l be an eigenvalue of $\bar{S}$. Then, because of Assumption 2, the matrix $\begin{bmatrix} A-\lambda I & b \\ c & 0\end{bmatrix}$ has full column rank. Thus, it follows from the structure of the matrix $\bar{S}-\lambda I$ that (13) has full column rank. On the other hand, let l be an eigenvalue of A but not of $\bar{S}$. Then, (13) still has full column rank since so does $\begin{bmatrix} A-\lambda I \\ c\end{bmatrix}$. □

Define $\Xi_i\in\mathbb{R}^{n+2m}$ by $\Xi=\begin{bmatrix}\Xi_1 & \Xi_2 & \cdots & \Xi_m\end{bmatrix}$ and let $\mu_i:=\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}\Xi_i$, $\mu:=\begin{bmatrix}\mu_1 & \mu_2 & \cdots & \mu_m\end{bmatrix}^T$, and $\chi_i:=\Xi_i-N_i\tilde{x}$, where $N_i:=\begin{bmatrix}0_{2i\times n} \\ I_n \\ 0_{2(m-i)\times n}\end{bmatrix}$. Then, it is seen that $\mu=\Xi^Tc_c^T$ and

$$\dot{\Xi}_i=(A_c-Lc_c)\Xi_i+(-a[i]e+b[i]u),$$

which, together with (8), implies

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$$\begin{aligned}\dot{\chi}_i &= (A_c - Lc_c)\Xi_i + (-a[i]e + b[i]u) - N_i\dot{\tilde{x}} \\ &= (A_c - Lc_c)\chi_i + ((A_c - Lc_c)N_i - N_iA - a[i]c)\tilde{x} \\ &\quad + (b[i] - N_ib)u + N_ib\bar{\gamma}\bar{w} \\ &= (A_c - Lc_c)\chi_i + N_ib\bar{\gamma}\bar{w}, \\ \mu_i &= \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}\chi_i + \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}N_i\tilde{x} \\ &= \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}\chi_i = c_c\chi_i.\end{aligned} \tag{14}$$

Thus, μ is bounded since the matrix $A_c - Lc_c$ is Hurwitz and $\bar{w} = T_e(\sigma)w$.

If some parts of the initial condition in (2) are zero, i.e., $\begin{bmatrix}w_i(0) & w_{i+1}(0)\end{bmatrix}^T = 0,\ i = 1,3,5,\cdots,2m-1$, the scalar variable $\bar{\gamma}\bar{w}$ in (14) does not contain m distinct signals (i.e., it is not sufficiently rich of order m [9, Definition 5.2.3]). In this case, it is difficult or even impossible to show that $\hat{\theta}$ converges to θ since the vector μ is not *persistence of excitation (PE)* [9, Theorem 5.2.1]. So instead of proving it, we introduce the following lemma.

Lemma 2: In the equation (5), $\hat{\theta}(t)$ is bounded and $\lim_{t\to\infty}\dot{\hat{\theta}}(t) = 0$ for any initial condition. ◇

Proof: Define $\tilde{\xi} := \hat{\xi} - \xi$, $\tilde{\theta} := \hat{\theta} - \theta$, and $\eta := \tilde{\xi} - \Xi\tilde{\theta}$. Then, we have

$$\begin{aligned}\dot{\tilde{\xi}} &= (A_c - Lc_c)\tilde{\xi} + \Xi\dot{\hat{\theta}} + \Psi\tilde{\theta}, \\ \dot{\tilde{\theta}} &= -K_{ad}\Xi^T c_c^T c_c \Xi\tilde{\theta} - K_{ad}\Xi^T c_c^T c_c\eta \\ &= -K_{ad}\mu\mu^T\tilde{\theta} - K_{ad}\mu\eta_1, \\ \dot{\eta} &= (A_c - Lc_c)\eta.\end{aligned} \tag{15}$$

Define the Lyapunov function candidate

$$V(t) = \tilde{\theta}^T(t)\tilde{\theta}(t) + \frac{K_{ad}}{2}\int_t^\infty \eta_1^2(\tau)d\tau.$$

Then, its time derivative according to (15) is given by

$$\begin{aligned}\dot{V} &= -2K_{ad}(\tilde{\theta}^T\mu)^2 - 2K_{ad}(\tilde{\theta}^T\mu)\eta_1 - \frac{K_{ad}}{2}\eta_1^2 \\ &= -2K_{ad}\left(\tilde{\theta}^T\mu + \frac{\eta_1}{2}\right)^2 \le 0.\end{aligned} \tag{16}$$

This implies that $V(t) \le V(0)$, and therefore, that $\tilde{\theta}$ and η_1 are bounded, i.e., $\hat{\theta}$ is bounded since $\tilde{\theta} = \hat{\theta} - \theta$ and θ is constant. To use Barbalat's lemma, let us check the uniform continuity of $\dot{V}$. The derivative of $\dot{V}$ is

$$\ddot{V} = -4K_{ad}\left(\tilde{\theta}^T\mu + \frac{\eta_1}{2}\right)\cdot\left(\dot{\tilde{\theta}}^T\mu + \tilde{\theta}^T\dot{\mu} + \frac{\dot{\eta}_1}{2}\right).$$

It is easy to check that $\ddot{V}$ is bounded since $\tilde{\theta}, \eta_1, \mu, \dot{\tilde{\theta}}, \dot{\mu}, \dot{\eta}_1$ are all bounded. Then, by Lemma 4.3 in [10], $\dot{V}$ converges to zero. Therefore, from (15) and (16), $\tilde{\theta}^T\mu$ tends to zero since η_1 converges to zero. Finally, by (15), $\dot{\tilde{\theta}}(t)\ (= \dot{\hat{\theta}}(t))$ converges to zero as time goes to infinity. □

Now, we present the proof of Theorem 1.

Proof of Theorem 1: With the help of (5) and (8), the equation (8) can be written as

$$\begin{aligned}\dot{\tilde{x}} &= A\tilde{x} - b\bar{\gamma}\bar{w} + b\begin{bmatrix}K & \bar{\gamma}\end{bmatrix}T_c^{-1}(\hat{\theta})\hat{\xi} \\ &= (A + bK)\tilde{x} - bK\tilde{x} - b\bar{\gamma}\bar{w} + b\begin{bmatrix}K & \bar{\gamma}\end{bmatrix}T_c^{-1}(\hat{\theta})(\xi + \tilde{\xi}) \\ &= (A + bK)\tilde{x} + b\begin{bmatrix}K & \bar{\gamma}\end{bmatrix}\left(-\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix} + T_c^{-1}(\hat{\theta})T_c(\theta)\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix}\right. \\ &\quad \left. + T_c^{-1}(\hat{\theta})\left(\eta + \sum_{i=1}^m(\chi_i + N_i\tilde{x})\tilde{\theta}_i\right)\right) \\ &= (A + bK)\tilde{x} + b\begin{bmatrix}K & \bar{\gamma}\end{bmatrix}\left(T_c^{-1}(\hat{\theta})\left(T_c(\theta) - T_c(\hat{\theta})\right)\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix}\right. \\ &\quad \left. + T_c^{-1}(\hat{\theta})\eta + T_c^{-1}(\hat{\theta})\sum_{i=1}^m\tilde{\theta}_i\chi_i + T_c^{-1}(\hat{\theta})\sum_{i=1}^m(\tilde{\theta}_iN_i)\tilde{x}\right).\end{aligned}$$

In the above equation, we have

$$T_c^{-1}(\hat{\theta})\left(T_c(\theta) - T_c(\hat{\theta})\right)\begin{bmatrix}\tilde{x} \\ \bar{w}\end{bmatrix} + T_c^{-1}(\hat{\theta})\sum_{i=1}^m(\tilde{\theta}_iN_i)\tilde{x} = 0$$

since $T_c(\theta) - T_c(\hat{\theta}) = -\begin{bmatrix}\bar{T}_c(\tilde{\theta}) & 0_{(n+2m)\times 2m}\end{bmatrix}$ and $\sum_{i=1}^m\tilde{\theta}_iN_i - \bar{T}_c(\tilde{\theta}) = 0$. Therefore, we obtain

$$\dot{\tilde{x}} = (A + bK)\tilde{x} + b\begin{bmatrix}K & \bar{\gamma}\end{bmatrix}\left(T_c^{-1}(\hat{\theta})\eta + T_c^{-1}(\hat{\theta})\sum_{i=1}^m\tilde{\theta}_i\chi_i\right).$$

Since $A + bK$ is Hurwitz, this system is ISS (input-to-state stable) [11]. Then, the state $\tilde{x}$ is bounded since η converges to zero in (15) and $\tilde{\theta}_i\chi_i$ is bounded for any $i = 1,2,\cdots,m$ by (14) and Lemma 2. Therefore, from Lemma 2, $\chi_i = \Xi_i - N_i\tilde{x}$, $\eta = \tilde{\xi} - \Xi\tilde{\theta}$, and $\xi = T_c(\theta)\begin{bmatrix}\tilde{x}^T & \bar{w}^T\end{bmatrix}^T$, all the states $x, \Xi, \xi, \hat{\xi}, \hat{\theta}$ of the overall closed-loop system are bounded.

Now we will show that the output error e converges to zero. From $\eta = \tilde{\xi} - \Xi\tilde{\theta}$ and $\mu = \Xi^T c_c^T$, we have

$$c_c\tilde{\xi} = c_c\eta + c_c\Xi\tilde{\theta} = c_c\eta + \mu^T\tilde{\theta}.$$

Then, since η and $\mu^T\tilde{\theta}$ converge to zero, we obtain, with (12),

$$c_c\tilde{\xi} = (c_c\hat{\xi} - e) = \hat{\xi}_1 - e = 0 \quad \text{as} \quad t \to \infty. \tag{17}$$

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Using similar arguments as in the equations (11) and (12) with $\hat{\bar{\xi}} := T_c(\hat{\theta})\left[\hat{\bar{x}}^T \quad \hat{\bar{w}}^T\right]^T$, we obtain

$$
\begin{aligned}
\begin{bmatrix} \dot{\hat{\bar{x}}} \\ \dot{\hat{\bar{w}}} \end{bmatrix} &= T_c^{-1}(\hat{\theta})\dot{\hat{\xi}} - T_c^{-1}(\hat{\theta})\dot{T}_c(\hat{\theta})\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} \\
&= T_c^{-1}(\hat{\theta})A_c T_c(\hat{\theta})\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} + T_c^{-1}(\hat{\theta})b_c u + T_c^{-1}(\hat{\theta})\Psi(e,u)\hat{\theta} \\
&\quad + T_c^{-1}(\hat{\theta})L(e - c_c\hat{\xi}) + T_c^{-1}(\hat{\theta})\Xi\dot{\hat{\theta}} - T_c^{-1}(\hat{\theta})\dot{T}_c(\hat{\theta})\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} \\
&= \begin{bmatrix} A & -b\bar{\gamma} \\ 0 & \bar{S} \end{bmatrix}\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix}u + T_c^{-1}(\hat{\theta})\left(\Psi(e,u) - \Psi(\hat{\bar{x}}_1,u)\right)\hat{\theta} \\
&\quad + T_c^{-1}(\hat{\theta})L(e - c_c\hat{\xi}) + T_c^{-1}(\hat{\theta})\Xi\dot{\hat{\theta}} - T_c^{-1}(\hat{\theta})\dot{T}_c(\hat{\theta})\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} \\
&= \begin{bmatrix} A + bK & 0 \\ 0 & \bar{S} \end{bmatrix}\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} \underbrace{-T_c^{-1}(\hat{\theta})\dot{T}_c(\hat{\theta})}_{M_1(t)}\begin{bmatrix} \hat{\bar{x}} \\ \hat{\bar{w}} \end{bmatrix} + \underbrace{T_c^{-1}(\hat{\theta})\Xi\dot{\hat{\theta}}}_{M_2(t)} \\
&\quad \underbrace{+ T_c^{-1}(\hat{\theta})\left(\Psi(e,u) - \Psi(\hat{\bar{x}}_1,u)\right)\hat{\theta}}_{M_3(t)} + \underbrace{T_c^{-1}(\hat{\theta})L(e - c_c\hat{\xi})}_{M_4(t)}.
\end{aligned} \tag{18}
$$

From the equation (18), we have

$$
\begin{aligned}
\dot{\hat{\bar{x}}} &= \left((A + bK) + \begin{bmatrix} I_n & 0_{n\times 2m} \end{bmatrix} M_1(t)\begin{bmatrix} I_n \\ 0_{2m\times n} \end{bmatrix}\right)\hat{\bar{x}} + \begin{bmatrix} I_n & 0_{n\times 2m} \end{bmatrix} \\
&\quad \cdot\left(M_1(t)\begin{bmatrix} 0_{n\times 2m} \\ I_{2m} \end{bmatrix}\hat{\bar{w}} + M_2(t) + M_3(t) + M_4(t)\right).
\end{aligned} \tag{19}
$$

Here, the states $\hat{\bar{x}}$ and $\hat{\bar{w}}$ are bounded since $T_c(\hat{\theta})$ is nonsingular for any $\hat{\theta}$ by Lemma 1. By virtue of $T_c(\hat{\theta})$, $\hat{\xi}_1 = \hat{\bar{x}}_1$ since the first row of $T_c(\hat{\theta})$ is $\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$. This implies that $\Psi(e,u) - \Psi(\hat{\bar{x}}_1, u)$ converges to zero by (17). Also, since $\dot{T}_c(\hat{\theta})$, $\dot{\hat{\theta}}$, $e - c_c\hat{\xi}$ converge to zero and $\hat{\theta}, \Xi$ are bounded, $M_1(t), \cdots, M_4(t)$ tend to zero. Then, by [11, Example 9.6], the systems (19) is ISS (input-to-state stable [11]) since the matrix $A + bK$ is Hurwitz. Therefore, the state $\hat{\bar{x}}$ converges to zero since the input to the system (19) decays to zero, which implies that $\lim_{t \text{®¥}} e(t) = 0$. □

A Numerical Example

Consider an unstable linear system

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} -1 & 1 \\ -2 & 0 \end{bmatrix}x + \begin{bmatrix} 0 \\ -3 \end{bmatrix}u + \begin{bmatrix} 1 & 0 & 2 & 0 \\ -1 & 0 & -3 & 0 \end{bmatrix}w, \\
e &= \begin{bmatrix} 1 & 0 \end{bmatrix}x + \begin{bmatrix} 2 & 0 & -1 & 0 \end{bmatrix}w,
\end{aligned}
$$

where the exogenous input w is generated by an exosystem

$$\dot{w} = \mathrm{diag}(\sigma_1 S_o, \sigma_2 S_o)w,$$

where the initial condition $\begin{bmatrix} w_1(0) & w_2(0) \end{bmatrix}^T = 0$ and $\begin{bmatrix} w_3(0) & w_4(0) \end{bmatrix}^T \neq 0$. Note that we suppose that the upper bound on the order of the systems is 4 ($= 2m$, $m = 2$), but the actual order is 2 since $\begin{bmatrix} w_1(0) & w_2(0) \end{bmatrix}^T = 0$ for all $t \geq 0$.

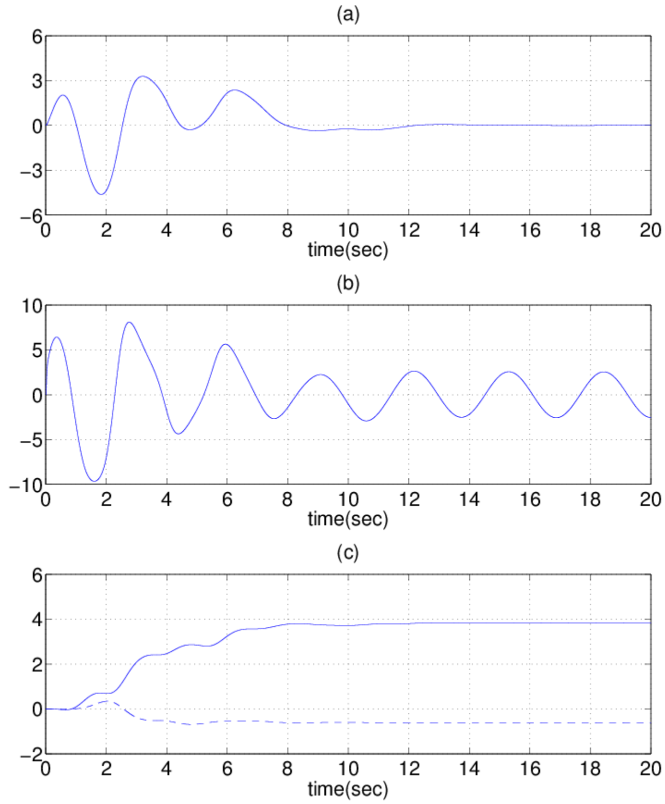

Figure 1. Simulation Results. (a) Output Error e, (b) Control Input u, (c) Estimated Values $\hat{\theta}_1$ (solid) and $\hat{\theta}_2$ (dashed).

Now, we design the proposed controller (5). The design parameters K, L, and K_{ad} are selected as

$$
\begin{aligned}
K &= \begin{bmatrix} 20.6 & 40.2 \end{bmatrix}, \\
L &= \begin{bmatrix} 102.8 & 381.7 & 652.9 & 650.2 & 374.2 & 100 \end{bmatrix}^T, \\
K_{ad} &= 1.0 \times 10^3.
\end{aligned}
$$

We carry out a computer simulation (Matlab/Simulink). For the simulation, let

$$\sigma_1 = 1 \quad \text{and} \quad \sigma_2 = 2.$$

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The simulation results are illustrated in Figure 1. As shown in Figure 1-(c), the estimated value $\hat{\theta}_1, \hat{\theta}_2$ are not correct. However, its time derivatives converge to zero. Also, as shown in Figure 1-(a), the output error e converges to zero.

Conclusions

We have presented a dynamic output feedback controller for known linear systems with unknown sinusoidal exogenous input representing the reference inputs and/or the disturbances. Although the proposed method requires the assumption that the plant does not have zeros, the controller has guaranteed that all the states of the overall closed-loop system are bounded and the output error converges to zero for any initial condition. In particular, it has been designed to be easy to implement under the only assumption that the upper bound on the order of the exosystem is known. In addition, it is simpler than the method proposed in [6] and does not involve any singularity problem for computation.

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