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AN APPROACH TO SNR IMPROVEMENT IN DISTRIBUTED BEAM FORMING OF RANDOMLY DISTRIBUTED SENSORS

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Abstract

The simultaneous transmission of data by more than one transmitter leads to interference in the destination. Though this can be overcome by introducing cooperative communication, still the SNR performance is poor. In this paper, Dual Beam Array (DBA) is considered to reduce interference in the place of two single beam arrays. The Dual Beam Array has the advantage of narrow beam width as it can be formed out of 2N sensors; each N sensors belonging to single beam array. In addition utilizing standard array geometries such as ULA, UCA and UEA better beam pattern is obtained. However, formation of standard arrays from randomly distributed sensors results in phase error. This is mitigated by estimating the array weight using LS Estimation. The simulation results of average beam pattern and SNR under large path loss channel model with Additive White Gaussian Noise (AWGN) show that the performance of Dual Beam Array is better than Two Single Beam Arrays transmitting simultaneously.

Keywords: Dual Beam Array, LS Estimation, Interference Reduction, Distributed Sensor Beamforming

1. Introduction

In recent times, there has been growing interest in the use of wireless sensor networks for monitoring field variables [1]. Sensor data need to be transmitted to the base station for further processing. Beamforming can be used to increase gain in the desired direction of the destination [2]. The feasibility, challenges and recent progress in this are discussed in [3] and [4]. The basic mechanism of beamforming is the phase and frequency synchronization among participating sensors. The performance characteristic of beamforming like main lobe gain, side lobe gain, beam width etc in the azimuthal pattern is analyzed for Uniform and Gaussian distributed sensors in [5], [6] and [7]. Different array geometries such as Uniform linear array (ULA), Uniform Circular array (UCA) and Uniform elliptical array (UEA) have advantages. Hence, it would be better to form ULA, UCA and UEA from randomly distributed sensors than considering a cluster of sensors.

Two approaches of distributed single beamforming for sensor networks are proposed in [8] and [9]. The first method uses the Euclidean distance and second one is using Least Square best fit line method to form Uniform Linear Array. As it is not possible to find actual nodes at the exact location of reference array, there exists a position error. The transmission equation with position error adjustment is given in [10].The phase perturbation caused by position error is derived and an iterative algorithm is proposed to solve the problem in [11] for data dependent beamforming. The position error has impact on the beam pattern though it does not have effect on the signal strength received in the intended destination. The phase errors caused by the position can be compensated by weight and the weights are estimated using LS Estimation technique [9]

When multiple arrays transmit data to different destinations, interference is caused. This problem can be combated at the receiver level by using appropriate filters or at the transmitter by using cooperative technique. However, the beam width of single beam array is wider resulting in poor SNR performance. Using a dual beam array in place of two cooperative single beam arrays, results in narrow beam width. Therefore, in this paper, a dual beam array is proposed for static channel with Additive White Gaussian Noise (AWGN) noise. Combining two single beam arrays of N elements, one dual beam array of 2N element is formed. The increased number of nodes reduces the beam width and increases the power density.

This proposed dual beam array is modeled as joint optimization of beam in two different directions and effect of position error minimization with single objective function. The solution is obtained by LS estimation. The distributed implementation of this technique is carried out HHT [9]. The SNR is computed at the destination assuming unity noise power for comparison purposes.

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This paper is organized as follows. The section 1 gives introduction and existing work. The section 2 explains the proposed work and Implementation. The section 3 presents Simulation and Result. The section 4 gives the conclusions.

2. Proposed Model

Fig 1 shows two single beam arrays simultaneously transmitting data to two different destinations. The signal from either array interferes with other due to the side lobe power. The side lobe power of array 2 interferes with main lobe power of array 1 and vice versa causing co-channel interference as shown in fig 1

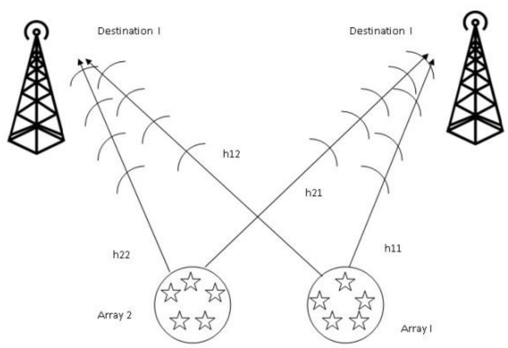

Fig 1 Interference due to Two Single Beam Arrays (TSBA)

The field strength at a point in space from the radiating element depends on the distance, the elevation angle and azimuthal angle and in terms of spherical coordinates system it is represented by AF(r,∅,θ). For easy understanding, the array gain is viewed as AF (θ), called elevation pattern and AF (∅), called azimuthal pattern. When the Cartesian position coordinates are included, the elevation pattern is represented as AF(θ|x,y) and the azimuthal pattern is given by AF(∅|x,y). But, for simplicity in representation, we are dropping the (x,y) in the representation here after in this paper. The average beam formed from N sensor array with the position of each element (x_i,y_i) is given by the equation (1) as in [12].

$$AF_{SBA}(\phi,\theta) = \frac{1}{N}\sum_{i=1}^{N} e^{jk(\delta_i-\varepsilon_i)} \quad (1)$$

Where $\delta_i = x_i \sin\theta\cos\phi + y_i \sin\theta\sin\phi$
$\varepsilon_i = x_i \sin\theta_0 \cos\phi_0 + y_i \sin\theta_0 \sin\phi_0$ $k = 2\pi/\lambda$ $\emptyset_0$ Azimuthal steer direction, θ_0 Elevation steer direction

2.1. Average SNR for Two Single Beam Arrays (TSBA)

In this section, the SNR at the destination using two single beam arrays of N/2 sensors is calculated. The received signal in any destination due to two single beam transmitting simultaneously given by equation (2) using equation (1)

$$r_{d,\text{TSBA}}(t) = \text{Re}\{\frac{m_1(t)\,e^{jw_c t}}{N/2}\sum_{i=1}^{N/2} e^{jk(\delta_{i1}-\varepsilon_{i1})} + \frac{m_2(t)\,e^{jw_c t}}{N/2}\sum_{i=1}^{N/2} e^{j\tau_i} e^{jk(\delta_{i1}-\varepsilon_{i1})}\} \quad (2)$$

Each symbol in message-1 and message-2 may be different or same. Considering the different symbols, the difference in symbol can be translated into phase delay assuming MPSK modulation and it denoted by τ. After substituting τ and rearranging equation (2), the following equation is obtained.

$$r_{d,\text{SBTA}}(t) = \text{Re}\{\frac{m(t)ej^{w_c t}}{N/2}(\sum_{i=1}^{N/2} e^{jk(\delta_{i1}-\varepsilon_{i1})} + e^{j\tau}\frac{1}{N/2}\sum_{i=1}^{N/2} e^{jk(\delta_{i2}-\varepsilon_{i2})})\} \quad (3)$$

Rearranging above equation (3), the average received power is given by following equation

$$P_{R,\text{TSBA}} = E[|\frac{m(t)ej^{w_c t}}{N/2}\frac{\sum_{i=1}^{N/2}(e^{jk(\delta_{i1}-\varepsilon_{i1})} + e^{jk(\delta_{i2}-\varepsilon_{i2})+\tau})}{2}|^2] \quad (4)$$

The steer angles are substituted in the variable δi in the above equation. Assuming the average message and noise power are unity, the average SNR is arrived as follows

$$SNR_{\text{TSBA}} = \frac{1}{N^2}E[|\sum_{i=1}^{N/2}(e^{jk\delta_{i1}} * w_{i1} + e^{jk\delta_{i2}+\tau} * w_{i2})|^2] \quad (5)$$

Where $w_{i1} = e^{jk\varepsilon_{i1}}, w_{i2} = e^{jk\varepsilon_{i2}}$. The above equation represents the normalized power gain of super position two arrays. The e^{τ} becomes 1 on expansion and taking modulus of equation. Hence, equation (9) can be rewritten as follows.

$$SNR_{\text{TSBA}} = \frac{1}{N^2}E[|\sum_{i=1}^{N/2}(e^{jk\delta_{i1}} * w_{i1} + e^{jk\delta_{i2}} * w_{i2})|^2] \quad (6)$$

From complex number property of modulus, it is found that the following relation holds for any two complex number a and b,

$$|a+b| \le |a| + |b| \quad (7)$$

It has been verified with simulation results and that the SNR is reduced in this case. The equation (6) shows that even with same message; the SNR reduces as per the equation (7).

2.2. Necessary condition for interference reduction in TSBA

The necessary condition the channel matrix $\begin{pmatrix} h_{11}, h_{12} \\ h_{21}, h_{22} \end{pmatrix}$, has to satisfy in order to reduce the interference can be obtained by solving the following optimization problem.

$$\min y^2 = (h_{11}x_1 - h_{21}x_2)^2 \quad (8)$$

The solution is given by the following equation

$$h_{11}x_1 = h_{21}x_2 \quad (9)$$

This necessary condition in equation (9) is not possible to achieve always. Because, the array-I calculates its weight with respect to base station I and vice versa. Therefore, in the other direction, the phase would not match which causes the phase of signals from neighbor array to be interfered. Though it is not possible to satisfy the condition in equation (9), an attempt is made to find out the probability of satisfying the above necessary condition in equation (13). There are four possible cases as explained below.

$$1) h_{11} = h_{21}, x_1 \neq x_2; 2) h_{11} \neq h_{21}, x_1 = x_2; 3) h_{11} = h_{21}, x_1 = x_2;$$
$$4) h_{11} \neq h_{21}, x_1 \neq x_2$$

The probability of satisfying the condition in equation (10) is given by the following.

$$P(h_{11} = h_{21})P(x_1 = x_2)P(h_{11} = h_{21}, x_1 = x_2) \leq \varepsilon \quad (10)$$

Where ε is very small number. Therefore, the Dual Beam Array is proposed in this paper.

2.3. SNR in Dual Beam Array (DBA)

The Dual Beam Array (DBA) is proposed to avoid the interference problem discussed in the previous section. The fig 2 shows the DBA transmitting data in two different directions.

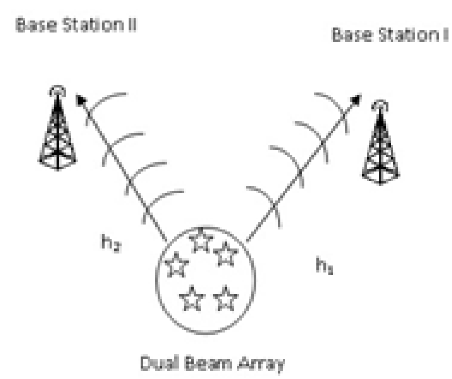

Fig 2 Dual Beam Array

The dual beams formed by array of N sensors are the superposition of individual response [14]. The average array factor in terms of position coordinates and steer angle can be obtained combining the equation 1 and response equation in [14]. The equation for dual beams array is given as

$$AF_{DBA}(\phi,\theta) = \frac{1}{N}\sum_{i=1}^{N}(e^{jk(\delta_i - \varepsilon_{i1})} + e^{jk(\delta_i - \varepsilon_{i2})}) \quad (11)$$

Where

$$\delta_i = x_i \sin\theta \cos\phi + y_i \sin\theta \sin\phi$$
$$\delta_i = x_i \sin\theta \cos\phi + y_i \sin\theta \sin\phi$$
$$\varepsilon_{i1} = x_i \sin\theta_1 \cos\phi_1 + y_i \sin\theta_1 \sin\phi_1$$
$$\varepsilon_{i2} = x_i \sin\theta_2 \cos\phi_2 + y_i \sin\theta_2 \sin\phi_2$$

After simplification, the equation (11) becomes

$$AF_{DBA}(\phi,\theta) = \frac{1}{N}\sum_{i=1}^{N} e^{jk\delta_i}(e^{jk\varepsilon_{i1}} + e^{jk\varepsilon_{i2}}) \quad (12)$$

The received signal modulated with message m (t) and carrier is given by equation (13)

$$r_{d,DBA}(t) = \mathrm{Re}\{m(t)e^{jw_c t}\sum_{i=1}^{N} e^{jk\delta i} * w_{Dual,i}\} \quad (13)$$

Where $w_{DBA,i} = \dfrac{e^{jk\varepsilon_{i1}} + e^{jk\varepsilon_{i2}}}{2}$

The average received average power can be estimated as follows

$$P_{R,DBA} = \frac{1}{N^2}E[| m(t)e^{jw_c t}\sum_{i=1}^{N} e^{jk\delta i} * w_{Dual,i} |^2] \quad (14)$$

The equation (22) gives the SNR assuming unity signal power and noise power.

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$$SNR_{DBA} = \frac{1}{N^2} E[\sum_{i=1}^{N} e^{jk\delta i} * w_{Dual,i} |^2] \quad (15)$$

2.4. Phase Error Minimization using LS Estimation

Standard array geometries have number of advantages. However, it is not always possible to form standard array geometries like ULA, UCA and UEA from randomly distributed sensor. The position error of the sensors has to be converted into phase error thereby causing the beampattern deviation from reference pattern, Hence, the antenna array weights have to be estimated to reduce the phase error variation. Let $AF_d(\emptyset)$ is the ideal reference array response and $AF(\emptyset)$ is the response of random array formed , then the error is the difference between desired response and calculated response. As the error is to be reduced, it becomes a minimization problem as given by equation

$$e = \min | AF_d(\phi) - AF(\phi) |^2 \quad (16)$$

Let D (∅) be the position matrix and W is the weight vector to be calculated. Each column of D (∅) corresponds to node and the row represents in azimuthal angle in discrete step. The solution is given by following equation [15].

$$W = D(\phi)^{-1} AF_d(\phi) \quad (17)$$

Similarly, it can be modeled with respect to θ

2.5. Distributed Algorithm

In centralized approach, the weight calculation is done by single sensor. This will deplete the battery life of that particular sensor. This weight is calculated using equation (17) by master node. To avoid the loading of single node, weight can be calculated by the distributed participating sensor. The QR decomposition is applied to matrix D (∅) with the help of House Holder Transformation (HHT) and the solution is obtained by back substitution as explained in [9]. This method has communication overhead of transmission of calculated weights and the coefficients to participating nodes. The flow chart in Fig 3 is for Distributed implementation of Weight calculation. The complexity of HHT is given as O (n^3). Though the complexity is the same as compared to centralized algorithm using Gauss elimination method, but the HHT requires more transmission of data among array elements.

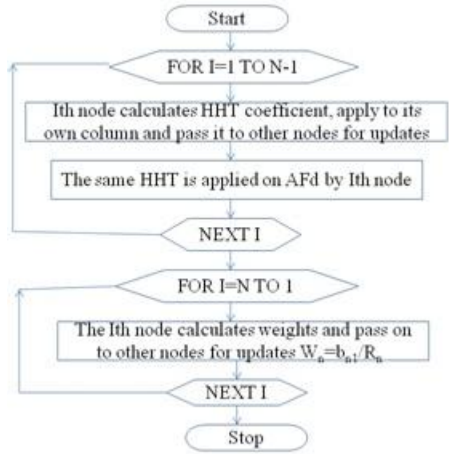


Fig 3 Flowchart of Distributed Beamforming Algorithm

3. Simulation results

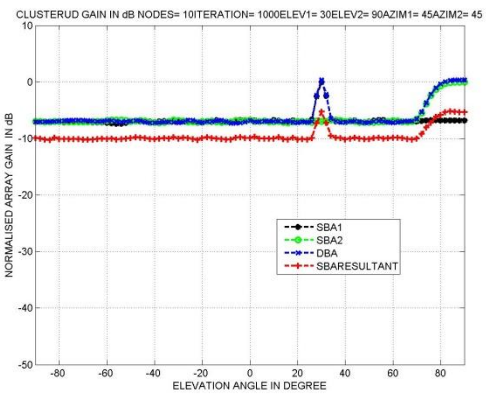
The ideal position of the sensors forming the ULA, UCA and UEA array was found. The uniform and Gaussian random perturbation was applied to each node separately. SNR in dB is calculated using equation (6) and (15). The simulation parameters are given in table 1. The weights are calculated from equation (17). The sensors are distributed in plane surface and the sensors positions are known. The sensors do not have motion. The number of array node is even. There is no reflection and scattering present. The sensors antenna is a isotropic radiator. One of the sensors can be assigned as master node which has higher capacity than rest of the nodes. Sensors are frequency synchronized.

SL NO	Parameters	Values
1	Frequency	2GHz
2	Array Type	ULA,UCA,UEA
3	Array Size(ULA,UCA,UEA)	10 Node
4	Iteration	1000
5	Distribution of Errors	Uniform & Gaussian
6	Perturbation Percentage (ULA, UCA, UEA)	10% of λ/2
7	Direction in degree	Elevation 30,90, Azimuthal 30,90

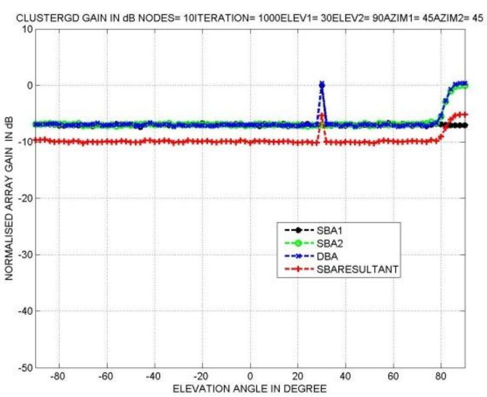
Table 1. Simulation Parameters

3.1. Gain of Cluster Array for same Azimuthal and different Elevation angles

The graphs in 4(a)&(b) show the array response of a cluster array formed from uniformly distributed sensors, cluster array formed from Gaussian distributed sensors. Each figure has 4 curves, Black for SBA1, Green for SBA 2, Red for Resultant of SBA 1 and 2 and Blue for DBA The simulation is done with elevation angle = 30^0, 90^0 and azimuthal angle 45^0. The DBA array gain response is better in the intended direction as compared to SBA Resultant.



4(a)



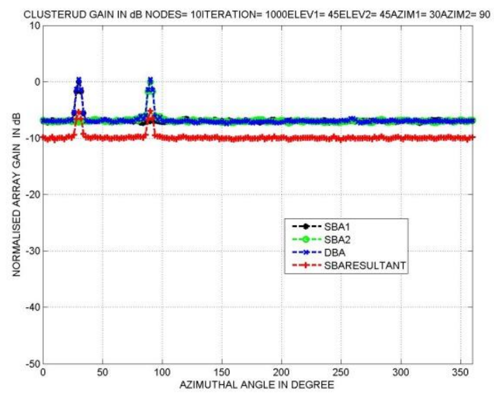
4(b)

Fig 4 Gain response of (a) Cluster UD, (b) Cluster GD, sensors in the direction of elevation angle=30^0 and 90^0 and azimuthal angles =45^0 .

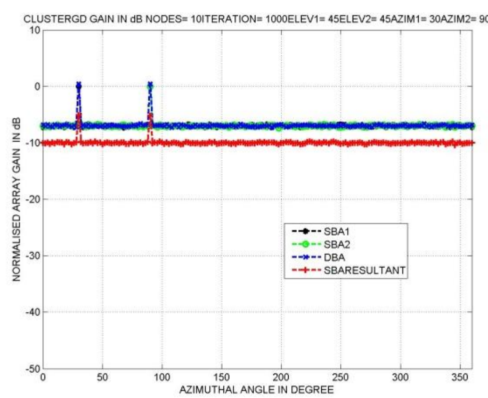
The side lobe level of dual beam array of the cluster UD and GD in Fig 4(a) & (b) is above -10dB. This would pose a serious problem for another receiver in the third destination as the signal from cluster array would interfere with that receiver

3.2. Array gain of Cluster Array with same Elevation and different Azimuthal angles

The results have been simulated for cluster Array for the case of same elevation and different azimuthal angles. The graph in 5(a) & (b) are the response of cluster Array formed from uniformly distributed sensors, Cluster Array formed from Gaussian distributed sensors. Each figure has 4 curves, Black for SBA1, Green for SBA 2, Red for Resultant of SBA 1 and 2 and Blue for DBA. The simulation is done with Azimuthal angle = 30^0, 90^0 and Elevation angle 45^0. Here once again, the DBA is better in the intended direction as compared to SBA resultant.



5(a)



5(b)

Fig 5 Gain response of (a) cluster UD, (b) cluster UD in the direction of Azimuthal angle=30^0 and 90^0 and Elevation angle =45^0.

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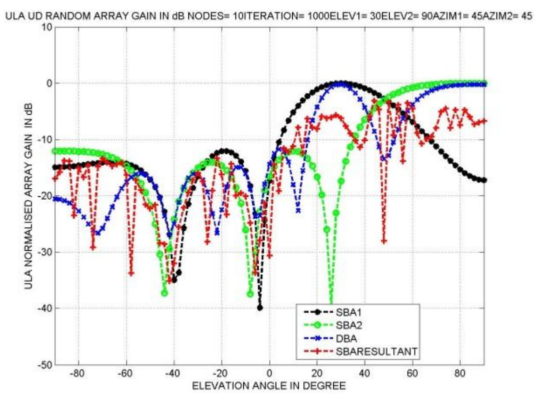
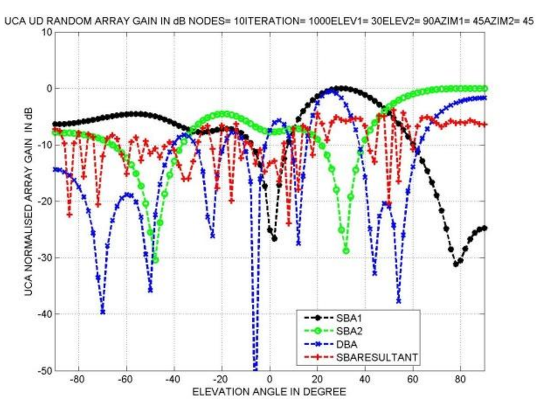
3.3. Array gain of a randomly perturbed Array for the same Azimuthal and different Elevation angles

The random perturbed array is formed in this way.The coordinates of ideal position are calculated first and then random perturbation using uniform and Gaussian variables are applied to it.

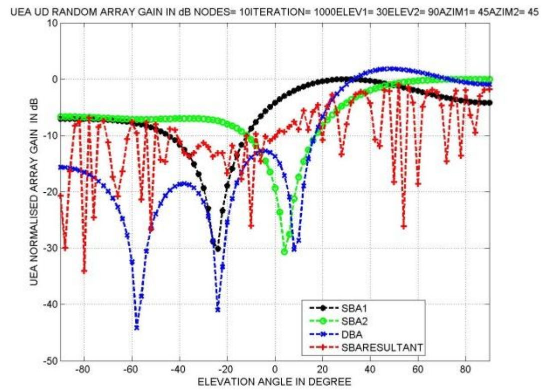
The graph in fig 6(a)-(c) are the Gain response of randomly perturbed (a) ULA, (b) UCA and (c) UEA array using uniformly distributed variables. Fig 6(d)-(f) are the SNR response of Gaussian variable perturbed (d) ULA, (e) UCA, (f) UEA arrays. Each figure has 4 acurves, Black for SBA1, Green for SBA 2, Red for Resultant of SBA 1 and 2 and Blue for DBA. The simulation is done with Elevation angle = 30^0, 90^0 and Azimuthal angle 45^0 and dual beams show better performance.



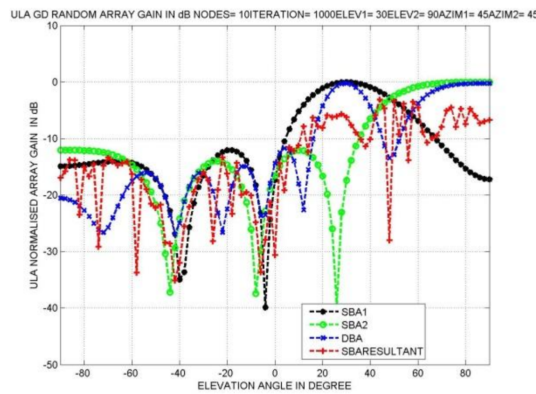
6(a)



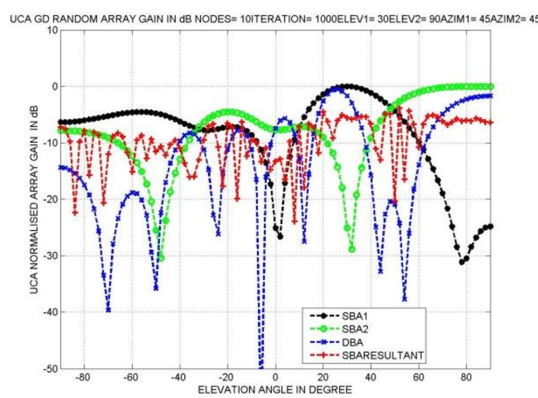
6(b)



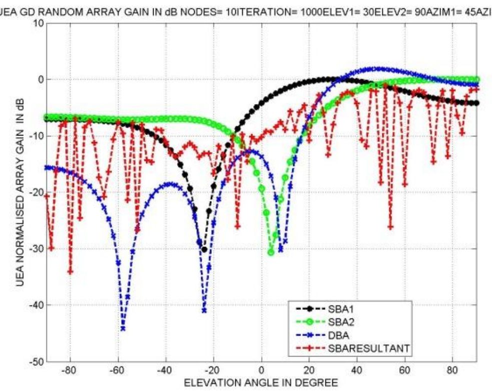
6(c)



6(d)



6(e)

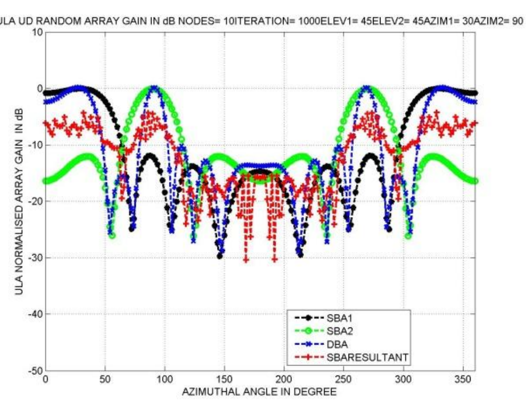


6(f)

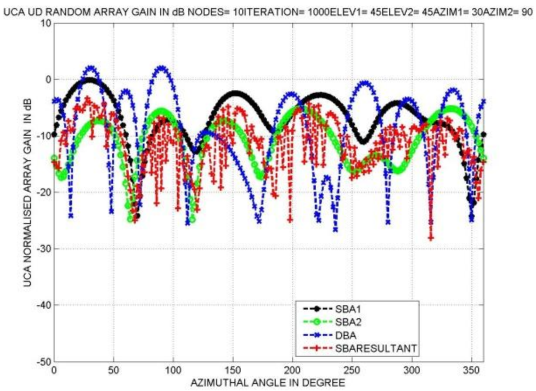
Fig 6 Gain response of (a) ULA UD, (b) UCA UD (c) UEA UD, (d) ULA GD, (e) UCA GD, (f) UEA GD in the direction of elevation angle=30^0 and 90^0 and azimuthal angle =45^0.

3.4. Array Gain of randomly perturbed array for same Elevation and different Azimuthal angles

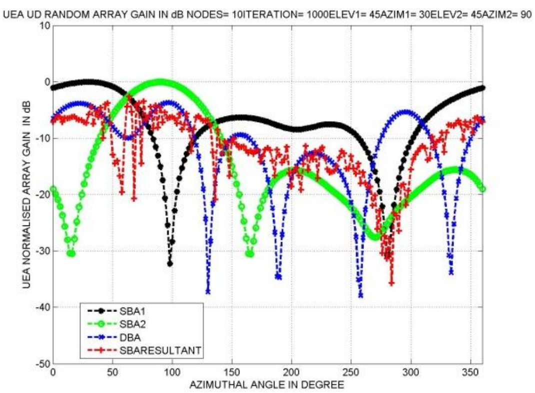
The graph in fig 7(a)-(c) are the Gain response of randomly perturbed (a) ULA, (b) UCA and (c) UEA array using uniformly distributed variables. Fig 7(d)-(f) are the response of Gaussian variable perturbed (d) ULA, (e)UCA,(f)UEA arrays. Each figure has 4 curves and same color as fig 7 is used. The simulation is done with Azimuthal angle = 30^0, 90^0 and Elevation angle 45^0.



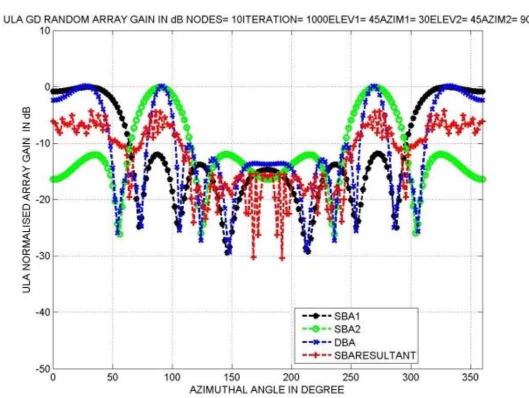
7(a)



7(b)



7(c)



7(d)

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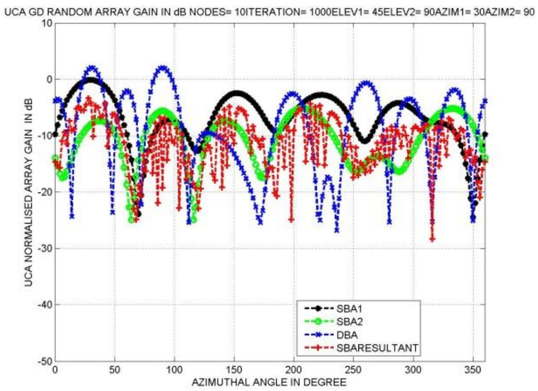
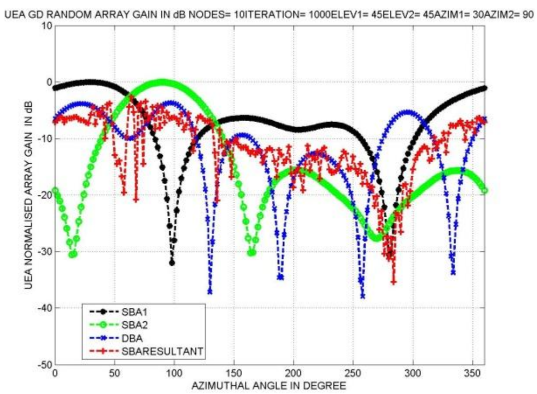


7(e)



7(f)

Fig 7 Gain response of (a) ULA UD, (b) UCA UD (c) UEA UD, (d) ULA GD, (e) UCA GD, (f) UEA GD in the direction of Azimuthal angle=30^0 and 90^0 and Elevation angle =45^0

3.5. SNR Performance of Dual Beam Array and Two Single Beam Array

The SNR for the dual beam array and the single beam are computed and are shown in the table 2 and 3. The SNR of Cluster array, ULA, UCA, UEA Reference array, perturbed ULA, UCA and UEA with uniform and Gaussian distributed sensors are presented in the elevation direction in table 2 and in azimuthal direction in table 3.

Table 2 Comparison of SNR for DBA and Two SBA in elevation directions I and II.

SLNO	Array Type	Two SBA Dir-I	Two SBA Dir-II	DBA Dir-I	DBA Dir-II
1	CLUS-TER UD	29.7284	29.4198	108.1906	108.1906
2	CLUS-TER GD	29.0339	30.4432	109.1894	109.1894
3	ULA UD	23.6998	21.3052	94.4559	94.5075
4	UCA UD	27.6258	22.6426	67.6322	67.6335
6	UEA UD	15.6145	65.4179	80.7237	80.7236
7	ULA GD	21.3151	21.3151	94.4450	94.5062
8	UCA GD	27.6172	22.6768	67.6326	67.6320
9	UEA GD	15.6114	65.3998	80.7236	80.7237

Table 3 Comparison of SNR of DBA and two SBAs in azimuthal directions I and II.

SLNO	Array Type	Two SBA Dir-I	Two SBA Dir-II	DBA Dir-I	DBA Dir-II
1	CLUSTER UD	29.3185	30.3187	109.4044	109.4044
2	CLUSTER GD	30.8848	30.1796	110.1425	110.1425
3	ULA UD	19.1409	35.1390	102.0905	101.7999
4	UCA UD	38.3757	1.1606	159.5171	159.6265
5	UEA UD	26.2222	34.3945	38.3731	38.3564
6	ULA GD	34.9946	34.9946	102.1705	101.7156
7	UCA GD	38.2262	1.1843	159.7596	160.0142
11	UEA GD	26.2879	34.3237	38.3061	38.2890

From the table2&3, it is observed, that the SNR of DBA in the destination directions 1 and 2 are higher than the resultant SNR of two SBA in the corresponding directions. To validate the results, the SNR of ULA, UCA and UEA arrays with sensors at the ideal positions, the reference array was simulated. It is also seen from the results that the perturbed array SNR is almost equal to reference array SNR for both SBA and DBA. The SNR is improved in the DBA because the interference is reduced. In addition, though the utilization of standard array geometries has not improved the SNR in the desired direction in both single beam and dual beam arrays compared to cluster of nodes, yet the side lobe levels are below -10 dB as seen in the figures 6 and 7, thereby reducing interference in the rest of the directions. In comparison to other geometries, the UCA geometry has the highest SNR performance for the dual beam array in the azimuthal direction.

4. Conclusion

Dual beam array to improve SNR in the multiple (dual) transmissions in distributed beamforming has been proposed in this paper. Simulation results indicate that the proposed method reduces the interference and improves the SNR in the intended destinations. With the same symbols transmitted in Two Single Beam Arrays (TSBA) simultaneously, interference occurs, thereby reducing the SNR. Though cooperative transmission between the N sensors can reduce interference, the lobe width is wider compared to a DBA using 2N Sensors. As the performance of DBA is independent of symbols and has narrow beam width, the DBA becomes best choice in the above scenario. The study also reveals that the use of standard array geometries like ULA, UCA and UEA further reduces the interference in the rest of the direction as compared to cluster of array. The DBA combined with LS estimation for position error minimization has reduced the interference in the other destinations, improving the overall system performance.

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