

RELIABLY ASSESSMENT FOR ROLLING MILLS USING ALGEBRA OF LOGICS

Pawan Kumar Sharma, Assistant Professor, Deptt. of Applied Science, Krishna Engineering College, Mohan Nagar, Ghaziabad, India

Ganesh Kr. Thakur, Assistant Professor, Deptt. of Applied Science, Krishna Engineering College, Mohan Nagar, Ghaziabad, India

Bandana Priya, Assistant Professor, Deptt. of Applied Science, G.L.Bajaj Institute of management & Technology, Gr. Noida, India

ABSTRACT:

In hot rolling, heated metal is passed between two rolls (taken as subsystem A and B) that rotate in opposite directions. The gap between the rolls being less than the thickness of entering metal. Because the rolls rotate with a surface velocity that exceeds the speed of the incoming metal, friction along the contact interface acts to propel the metal forward. The metal is squeezed and elongates to compensate for the decreased in cross-sectional area. The amount of deformation that can be achieved in a single pass between a given pair of rolls depends on the friction conditions along the interface, In too much is demanded, the rolls can not advance the material and simply skid over its surface. Too little deformation per pass, however, results in excessive production cost.

In this present discussion, two rolls i.e. upper roll and lower roll are depicted as subsystem A and B , and further categorized as main system (A_1, B_1) and its standby redundant (A_2, B_2) , connected in series respectively. Redundancy is particularly valuable where it is not possible to do maintenance and of course one of the method of increasing reliability. There are two standby redundant generators G_1 and G_2 used to supply power to rolling mills and these can be connected through the change over device COD. The system has switching devices E_{12} and E_{21} . To utilize survival subsystem in A_i and B_i ($i = 1, 2$) a switching device E connecting main and standby system is being used in addition and the system is called a compound redundant system. Here "compound" word means to have switching devices in order to achieve higher system's reliability. Boolean function technique has been used to formulate mathematical model of the system. For the system presented here, Algebra of logics is being used to obtain reliability and M.T.T.F. of the considered system. A numerical illustration to detect the effects of the compound system to the usual standby is examined.

KEYWORDS:

Boolean function, Algebra of logics, Performance measures, Reliability, M.T.T.F etc.

1. INTRODUCTION:

In modern industries, systems are designed to be operative for a specified period (called the mission time), i.e. there should be no failure in any equipment or part of equipment under specified operating conditions during the total period (operative time, administrative time and repair time).

Behaviour analysis of each item of equipment under given operating conditions is helpful to design the component for minimum failure and to prepare a plan in advance to scheduled preventive maintenance.

Although the basic concepts of many forming processes have remained largely unchanged throughout history, the details and equipment have evolved considerably. Manual processes were converted to machine processes during the industrial revolution. The machinery then became bigger, faster and more powerful. Water wheel power was replaced by steam and then electricity. More recently, computer controlled, automated operations have emerged. Processes that are normally performed "hot", is presented in this paper.

An obvious reason for the popularity of the hot-working processes is that they often provide an attractive means of producing a desired shape. At elevated temperature, metals weaken and become more ductile. With continual recrystallization, massive deformation can take place without exhausting material plasticity. In steels, hot forming involves the deformation of weaker, austenite structure, which then cools to the stronger, room-temperature, ferrite or much stronger non-equilibrium structures.

Among the hot-working processes viz, rolling, forging, extrusion, hot drawing, pipe welding and piercing, rolling is of major importance in modern manufacturing.

Rolling is usually the first process that is used to convert material into a finished wrought product. Thick starting stock can be rolled into blooms, billets or slabs or these shapes can be obtained directly from continuous casting. A *bloom* has square or rectangular cross section, with a thickness greater than 6 inches and a width no greater than twice the thickness. A *billet* is usually smaller than a bloom and has a square or circular cross section. Same form of deformation process, such as rolling or extrusion, usually produces billets. A *slab* is a rectangular solid where the width is greater than twice the thickness. Slabs can be further rolled to produce plate, sheet and strip. These hot-worked products often form the starting material for subsequent processing using techniques such as cold forming or machinery. Sheet and strip can be fabricated into products or further cold rolled into thinner, stronger material or even into foil. Blooms are billets and can be further rolled into finished product such as structural shapes or railroad into finished product, such as structural shapes or railroad rail or they can be processed into semi-finished shapes, such as bar, rod, tube or pipe.

Fig-1 shows the logical block diagram of considered system during this study. The following assumptions have been used during the study:

1. The system consists of two main units A_1, B_1 and two standby units A_2, B_2 connected in series, respectively.
2. At $t = 0$, A_1 and B_1 start operating while A_2 and B_2 are kept as standby.
3. None of the standby units degrade in unused condition.
4. The system ceases to function when both the units of any subsystem are non-operative.
5. Units are non-identical and statistically independent.
6. The system has two modes of states, viz; normal and failed.
7. Switching devices used are imperfect.
8. All failures are exponentially distributed.

The following notations have been used for mathematical modeling of the system:

- x_1, x_3 : States of generator
- x_2 : State of change over device.
- x_4, x_5 : States of main rolls A_1 and B_1 .
- x_7, x_8 : States of standby rolls A_2 and B_2 .
- x_6, x_9, x_{10} : States of switching devices E, E_{12}, E_{21} , respectively.
- x'_i : Negation of x_i , $\forall i = 1, 2, \dots, 10$.
- \wedge / \vee : Conjunction / Disjunction
- R_i : Reliability of i^{th} state, $\forall i = 1, 2, \dots, 10$.
- x_i : $\begin{cases} 1, \text{ good state} \\ 0, \text{ bad state} \end{cases} \forall i = 1, 2, \dots, 10$.

λ_i : Failure rate of i^{th} component of the system.

2. MATERIAL AND METHODS:

This study was conducted at Dept. of Mathematics, K.E.C., Sahibabad, Ghaziabad, India during April 2015. The results obtained are studied there during June 2015.

In this study, the author has been used Boolean function technique (Gupta, P.P. et al, 1983) to formulate mathematical model of the considered system. Various paths of successful operation of the system have been obtained. The reliability of considered system and MTTF (Zhimin He. et. al. 2005) has been evaluated. These results can be used to obtain various

reliability parameters of the system having similar configurations.

Using Boolean function technique, the conditions of capability of the successful operation of complex system in terms of logical matrix are expressed as:

$$F(x_1, x_2, \dots, x_{10}) = \begin{bmatrix} x_1 & x_2 & x_4 & x_5 & \\ x_1 & x_2 & x_4 & x_8 & x_9 \\ x_1 & x_2 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x_6 & x_7 & x_8 \\ x_2 & x_3 & x_4 & x_5 & \\ x_2 & x_3 & x_4 & x_8 & x_9 \\ x_2 & x_3 & x_5 & x_7 & x_{10} \\ x_2 & x_3 & x_6 & x_7 & x_8 \end{bmatrix}$$

... (1)

3. SOLUTION OF THE MODEL:

By application of algebra of logical, equation (1) may be written as

$$F(x_1, x_2, \dots, x_{10}) = [x_2 \wedge f] \text{ ----- (2)}$$

$$\text{where, } f = \begin{bmatrix} x_1 & x_4 & x_5 & \\ x_1 & x_4 & x_8 & x_9 \\ x_1 & x_5 & x_7 & x_{10} \\ x_1 & x_6 & x_7 & x_8 \\ x_3 & x_4 & x_5 & \\ x_3 & x_4 & x_8 & x_9 \\ x_3 & x_5 & x_7 & x_{10} \\ x_3 & x_6 & x_7 & x_8 \end{bmatrix} \text{ -----(3)}$$

Now substituting the following in equation (3):

$$A_1 = [x_1 \ x_4 \ x_5] \text{ -----(4)}$$

$$A_2 = [x_1 \ x_4 \ x_8 \ x_9] \text{ -----(5)}$$

$$A_3 = [x_1 \ x_5 \ x_7 \ x_{10}] \text{ -----(6)}$$

$$A_4 = [x_1 \ x_6 \ x_7 \ x_8] \text{ -----(7)}$$

$$A_5 = [x_3 \ x_4 \ x_5] \text{ -----(8)}$$

$$A_6 = [x_3 \ x_4 \ x_8 \ x_9] \text{ -----(9)}$$

$$A_7 = [x_3 \ x_5 \ x_7 \ x_{10}] \text{ ... (10)}$$

$$A_8 = [x_3 \ x_6 \ x_7 \ x_8] \text{ ... (11)}$$

We obtain

$$f = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \end{bmatrix} \dots\dots\dots(12)$$

Using orthogonalisation algorithm, equation (12) may be written as:

$$f = \begin{bmatrix} A_1 & & & & & & & & \\ A_1' & A_2 & & & & & & & \\ A_1' & A_2' & A_3 & & & & & & \\ A_1' & A_2' & A_3' & A_4 & & & & & \\ A_1' & A_2' & A_3' & A_4' & A_5 & & & & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6 & & & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6' & A_7 & & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6' & A_7' & A_8 & \end{bmatrix} \dots\dots\dots(13)$$

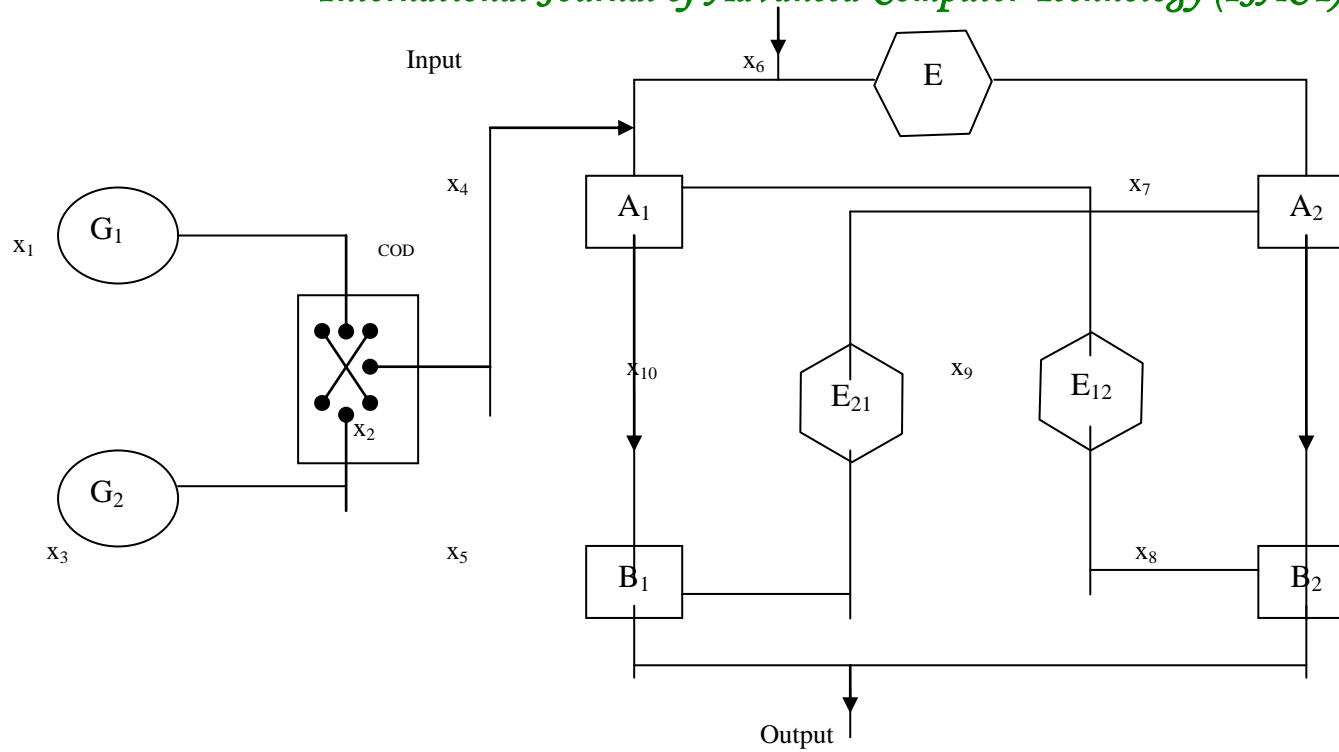


Fig-1: Logical Block Diagram

By algebra of logics, we compute the following

$$A'_1 = \begin{bmatrix} x'_1 \\ x_1 & x'_4 \\ x_1 & x_4 & x'_5 \end{bmatrix}$$

$$\begin{aligned} \therefore A'_1 A_2 &= \begin{bmatrix} x'_1 \\ x_1 & x'_4 \\ x_1 & x_4 & x'_5 \end{bmatrix} \wedge [x_1 & x_4 & x_8 & x_9] \\ &= \begin{bmatrix} x_1 & x_4 & x'_5 & x_8 & x_9 \end{bmatrix} \end{aligned} \quad \dots(14)$$

Similarly, we obtain

$$A'_1 A'_2 A_3 = [x_1 \quad x'_4 \quad x_5 \quad x_7 \quad x_{10}] \quad \dots(15)$$

$$A'_1 A'_2 A'_3 A_4 = [x_1 \quad x'_4 \quad x'_5 \quad x_6 \quad x_7 \quad x_8] \quad \dots(16)$$

$$A'_1 A'_2 A'_3 A'_4 A_5 = [x'_1 \quad x_3 \quad x_4 \quad x_5] \quad \dots(17)$$

$$A'_1 A'_2 A'_3 A'_4 A'_5 A_6 = [x'_1 \quad x_3 \quad x_4 \quad x'_5 \quad x_8 \quad x_9] \quad \dots(18)$$

$$A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A_7 = [x'_1 \quad x_3 \quad x'_4 \quad x_5 \quad x_7 \quad x_{10}] \quad \dots(19)$$

$$A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A_8 = [x'_1 \quad x_3 \quad x'_4 \quad x'_5 \quad x_6 \quad x_7 \quad x_8] \quad \dots(20)$$

Using equation (14) through (20), equation (13) becomes

$$f = \begin{bmatrix} x_1 & x_4 & x_5 \\ x_1 & x_4 & x'_5 & x_8 & x_9 \\ x_1 & x'_4 & x_5 & x_7 & x_{10} \\ x_1 & x'_4 & x'_5 & x_6 & x_7 & x_8 \\ x'_1 & x_3 & x_4 & x_5 \\ x'_1 & x_3 & x_4 & x'_5 & x_8 & x_9 \\ x'_1 & x_3 & x'_4 & x_5 & x_7 & x_{10} \\ x'_1 & x_3 & x'_4 & x'_5 & x_6 & x_7 & x_8 \end{bmatrix} \quad \dots(21)$$

Putting the value of f from equation (21) in equation(2),

$$F(x_1, x_2, \dots, x_{10}) = \begin{bmatrix} x_1 & x_2 & x_4 & x_5 \\ x_1 & x_2 & x_4 & x'_5 & x_8 & x_9 \\ x_1 & x_2 & x'_4 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x'_4 & x'_5 & x_6 & x_7 & x_8 \\ x'_1 & x_2 & x_3 & x_4 & x_5 \\ x'_1 & x_2 & x_3 & x_4 & x'_5 & x_8 & x_9 \\ x'_1 & x_2 & x_3 & x'_4 & x_5 & x_7 & x_{10} \\ x'_1 & x_2 & x_3 & x'_4 & x'_5 & x_6 & x_7 & x_8 \end{bmatrix} \quad \dots(22)$$

Since, R.H.S. of equation (22) is disjunction of pair-wise disjoint conjunctions, therefore the reliability of considered system as a whole is given by:

$$\begin{aligned} R_S &= \Pr\{F(x_1, x_2, \dots, x_{10}) = 1\} \\ &= R_2 [R_1 R_4 R_5 + S_5 R_1 R_4 R_8 R_9 + S_4 R_1 R_5 R_7 R_{10} + S_4 S_5 R_1 R_6 R_7 R_8 \\ &\quad + S_1 R_3 R_4 R_5 + S_1 S_5 R_3 R_4 R_8 R_9 + S_1 S_4 R_3 R_5 R_7 R_{10} + S_1 S_4 S_5 R_3 R_6 R_7 R_8] \end{aligned}$$

where, R_i is the reliability corresponding to system state x_i while $S_i = 1 - R_i$, $\forall i = 1, 2, \dots, 10$.

Thus, we have

$$\begin{aligned} R_S &= R_2 [R_1 R_4 R_5 + R_3 R_4 R_5 + R_1 R_4 R_8 R_9 + R_1 R_5 R_7 R_{10} + R_1 R_6 R_7 R_8 + R_3 R_4 R_8 R_9 \\ &\quad + R_3 R_5 R_7 R_{10} + R_3 R_6 R_7 R_8 + R_1 R_4 R_5 R_6 R_7 R_8 + R_1 R_3 R_4 R_5 R_8 R_9 \\ &\quad + R_1 R_3 R_4 R_5 R_7 R_{10} + R_1 R_3 R_4 R_6 R_7 R_8 + R_1 R_3 R_5 R_6 R_7 R_8 \\ &\quad + R_3 R_4 R_5 R_6 R_7 R_8 - R_1 R_3 R_4 R_5 - R_1 R_4 R_5 R_8 R_9 - R_1 R_4 R_5 R_7 R_{10} \\ &\quad - R_1 R_4 R_6 R_7 R_8 - R_1 R_5 R_6 R_7 R_8 - R_1 R_3 R_4 R_8 R_9 - R_3 R_4 R_5 R_8 R_9 - R_1 R_3 R_5 R_7 R_{10} \\ &\quad - R_3 R_4 R_5 R_7 R_{10} - R_1 R_3 R_6 R_7 R_8 - R_3 R_4 R_6 R_7 R_8 - R_3 R_5 R_6 R_7 R_8 \\ &\quad - R_1 R_3 R_4 R_5 R_6 R_7 R_8] \end{aligned} \quad \dots(23)$$

4. SOME PARTICULAR CASES:

CASE I: When reliability of each component is R:

In this case, setting $R_i (i = 1, 2, \dots, 10) = R$ in equation (23), we obtain

$$R_S = 2R^4 + 5R^5 - 11R^6 + 6R^7 - R^8 \quad \dots(24)$$

^{we} **CASE II: When failure rates follow Weibull time distribution:**

Let λ_i be the failure rate corresponding to system state x_i , then in this case, reliability of that component is given by

$$R_i(t) = e^{-\lambda_i t^\alpha}, \text{ for all } i = 1, 2, \dots, 10$$

where α is some positive parameter.

Using above results in equation (23), we obtain the reliability of considered system at time instant t as:

$$R_{SW}(t) = \sum_{i=1}^{14} \exp\{-a_i t^\alpha\} - \sum_{j=1}^{13} \exp\{-b_j t^\alpha\} \quad \dots(25)$$

where,

- $a_1 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_2$
- $a_2 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$
- $a_3 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_8 + \lambda_9$
- $a_4 = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $a_5 = \lambda_1 + \lambda_2 + \lambda_6 + \lambda_7 + \lambda_8$
- $a_6 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_8 + \lambda_9$
- $a_7 = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $a_8 = \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 + \lambda_8$
- $a_9 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$
- $a_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_8 + \lambda_9$
- $a_{11} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $a_{12} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8$
- $a_{13} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$
- $a_{14} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$
- $b_2 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_8 + \lambda_9$
- $b_3 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $b_4 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_5 = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_6 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_8 + \lambda_9$
- $b_7 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_8 + \lambda_9$
- $b_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $b_9 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_{10}$
- $b_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_{11} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_{12} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$
- $b_{13} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8$

Case III: When failure rates follow exponential time distribution:

Exponential distribution is a particular case of Weibull distribution for $\alpha = 1$ and is much useful in numerous practical problems. The reliability of the system in this case at any instant 't', can be obtained by putting $\alpha = 1$ in equation (25), and is

$$R_{SE}(t) = \sum_{i=1}^{14} \exp\{-a_i t\} - \sum_{j=1}^{13} \exp\{-b_j t\} \quad \dots(26)$$

Also, an important reliability parameter viz; M.T.T.F., in this case, is given by

$$M.T.T.F = \int_0^\infty R_{SE}(t) dt = \sum_{i=1}^{14} \left(\frac{1}{a_i}\right) - \sum_{j=1}^{13} \left(\frac{1}{b_j}\right) \quad \dots(27)$$

where, a_i 's and b_j 's have been mentioned earlier.

5. NUMERICAL ILLUSTRATION:

For a numerical illustration, consider the following:

(i) $\lambda_i = 0.3 \quad (\forall i = 1, 2, \dots, 9)$ and $\alpha = 2$ in equation (25);

(ii) $\lambda_i = 0.3 \quad (\forall i = 1, 2, \dots, 9)$ in equation (26);

and (iii) $\lambda_i (i = 1, 2, \dots, 9) = \lambda = 0, 0.1, 0.2, \dots, 1.0$, in equation (27);

One can compute the *table-1* and 2. Corresponding graphs have been shown in *fig-2* and 3, respectively.

Table-1: Values of Reliability at different time

t	R _{SW} (t)	R _{SE} (t)
0	1	1
1	0.481004	0.481004
2	0.009401	0.149245
3	5.54 x 10 ⁻⁶	0.038985
4	1.51 x 10 ⁻¹⁰	0.009401
5	2.07 x 10 ⁻¹⁶	0.002178
6	1.41 x 10 ⁻²³	0.000494
7	4.8 x 10 ⁻³²	0.000111
8	8.12 x 10 ⁻⁴²	2.48 x 10 ⁻⁵
9	6.84 x 10 ⁻⁵³	5.54 x 10 ⁻⁶
10	2.87 x 10 ⁻⁶⁵	1.23 x 10 ⁻⁶

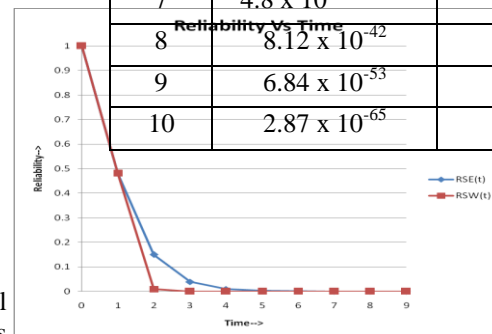


Fig-2: Reliability Vs Time

λ	M.T.T.F.
0.0	∞
0.1	3.45635
0.2	1.72817
0.3	1.15212
0.4	0.86409
0.5	0.69127
0.6	0.57606
0.7	0.49376
0.8	0.43204
0.9	0.38404
1.0	0.34563

Table-2: Values of MTTF at different failure rate

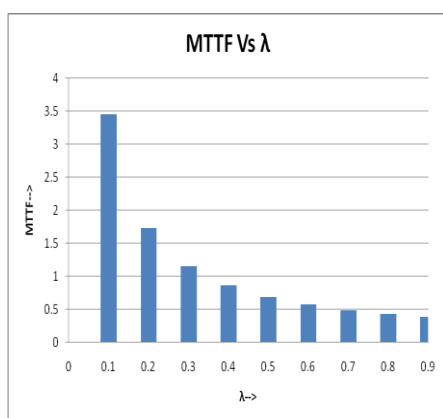


Fig-3: MTTF Vs Failure rate Lamda

6. RESULTS AND DISCUSSION:

In this study, the author has analyzed a rolling mills system for its performance measurement with the aid of Boolean function technique and algebra of logics. Boolean function technique has been used to formulate mathematical model of the system. For the system presented here, Algebra of logics is being used to obtain reliability and M.T.T.F. of the considered system. A numerical illustration to detect the effects of the compound system to the usual standby is examined.

Reliability of the system as a whole has obtained in three different cases, M.T.T.F. of the system has also obtained. Graphical illustration followed a numerical example has appended at last to highlight important results of the study.

A critical examination of table -1 & fig - 2 reveals that reliability of the system decreases rapidly in case of failures follow Weibull time distribution but it decreases in a smooth

and better way when failures follow exponential time distribution.

Study of table-2 & fig-3 yields that the M.T.T.F. of the system decreases catastrophically in the beginning but thereafter it decreases approximately in a constant manner.

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