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Mean Queue Size in a Queue With Discrete Autoregressive Gaussian Process Arrivals of Order P

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Abstract: *The class of Gaussian processes is one of the most widely used families of stochastic processes for modeling dependent data observed over time, or space, or time and space. The popularity of such processes stems primarily from two essential properties. First, a Gaussian process is completely determined by its mean and covariance functions. This property facilitates model fitting as only the first- and second-order moments of the process require specification. Second, solving the prediction problem is relatively straightforward. The best predictor of a Gaussian process at an unobserved location is a linear function of the observed values and, in many cases; these functions can be computed rather quickly using recursive formulas. We consider a discrete time dual server queuing system queuing networks with negative customers, signals, triggers where they arrive Gaussian process of order p (DAR(P)/D/s, and the service time of a customer is one slot. In contrast with the normal positive customers, negative customers arriving to a non-empty queue remove and work from the queue For this queuing system, we give an expression for the mean queue size. Further we propose approximation methods for the mean queue size which is based on matrix method.*

Key Words: Gaussian process, mean queue size, discrete time

1 Introduction

Gelenbe introduced a new class of queuing networks with two types of customers. The first type of customers is regular customers and they are treated in the normal way by a server.

We call these customers positive or regular customers. A positive customer obeys the specified Service and routing disciplines that determine the dynamic of the network under consideration. On the other hand, the second type of negative customers have the effective of a signal which induces a positive customer in the node, if any, to leave immediately the node.Thisqueueing network with positive and negative customers was initially motivated by neural network modeling. In this context, a node represents a neuron. Positive and negative customers routing in the network represent excitation and inhibition signals which increase or reduce in one unit the potential of the neuron to which they arrive. Extensions of the original network of Gelenbelead to a versatile class called in the literature as G-networks because it provides a unifying basis for queuing and neural networks. This analogy was discussed in detail in the survey paper by Gelenbe.

The time-series models with relatively few parameters are well suited for 'accurate and meaningful' modeling of various traffic sources in high-speed applications. Several queuing systems with time-series models have been analyzed and number of results has been analyzed and a number of results have been investigated by many authors. Investigated by many authors. In the continuous time case, Finch and Pearce (1965) andPearce (1966) considered an MA/M/1 queuing system, which has a moving average processas an input process and an exponential service time. In the cases of moving average processes of orders 1 and 2 for the inter arrival times, explicit expressions for the tail behavior of the queue size distribution were provided. The approach of Finch and Pearce can be extended to any finite order moving average model, although the complexity of computations increases exponentially with the order of the moving average model. For a discrete time queuing system with discrete moving average process of order 1 as an input process, He and Sohraby (2003) obtained a simple closed-form expression for the stationary distribution of the queue size. Addie and Zukerman (1994) provided an approximation in a closed form for the stationary virtual waiting time distribution in the discrete time queuing system with an arrival process that follows an autoregressive process of order 1 and constant service time. In this paper, we study the discrete time queuing model with a discrete autoregressive process of order p (DAR(p)) as an input process. The DAR(p) process, constructed and analyzed by Jacobs and Lewis (1978), has developed into one of several standard tools for modelling input traffic in telecommunication networks. Elwalid et al. (1993) and Heyman et al. (1992) analyzed several traces of variable-bit-rate videoconferencing traffic and model led them using the

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DAR(1) process. Ryu and Elwalid(1996) used the DAR(p) process to accurately model video traffic. In addition, Widjaja and Elwalid(1999) used theDAR(p) process as an input process to compare the performance between virtual circuits
(VC) merging and non-VC merging. Besides these works, the DAR(p) process has also been used in other research areas. For example, Dehnert et al. (2005) used theDAR(p)process to study the correlation structures of DNA sequences.
To the best of our knowledge, there are no analytic results on queues with DAR(p) arrivals, except for the case of p = 1. For a discrete time single sever queue with DAR(1)inputs, Hwang and Sohraby(2003)and Wang et al. (2002)derived the probability generating functions of the stationary queue size and the stationary waiting time, respectively. Kim
et al. (2007) obtained the stationary distributions of the queue size and the waiting time in transform-free form using an embedded Markov chain and the Bernoulli arrivals see time averages property. For a discrete time multi server queue with DAR(1) inputs, Choi et al.(2004) obtained the stationary distributions of the queue size and the waiting time using the matrix analytic method. For a discrete time single server queue with DAR(1) input, Kim and Sohraby(2006)investigated the tail behaviors of the queue size and the waiting time distributions. In the current paper, for the DAR(p)/D/2 queue, we give an expression for the mean queue size, which yields upper and lower bounds for the mean queue size. Further we propose one approximation methods for the mean queue size. which is based on the matrix analytic method . We show, by illustrations, that the proposed approximations are very accurate and computationally efficient. The remainder of this paper is organized as follows.
Section 2provides an exposition ofDAR(p) process and contains our model.
. In Sect. 3. The priority positive and negative queuing discipline and PGF of queue size.

In Sect.4 we propose approximation methods for the mean queue size, and illustrate that the approximations are very accurate and computationally efficient .

2 DAR(p)/D/2queueing model

Let $\{B(t) : t = . . . ,−1, 0, 1, . . .\}$ be a sequence of independent and identically distributed random variables taking the non negative integer values .A discrete autoregressive process of order 1 (DAR(1)) $\{A(t) : . . . ,−1, 0, 1, . . .\}$ is defined by the regression equation$(t) = (1−\alpha(t))A(t −1) +\alpha(t)B(t), \ t = . . . ,−1, 0, 1, . . . ,$ *(1)*
where $\{\alpha(t) : t = . . . ,−1, 0, 1, . . .\}$ is a Bernoulli process with $P(\alpha(t) = 0) = \delta\ (0 < \delta <1)$ and $P(\alpha(t) = 1) = 1 − \delta$, that is independent of $\{B(t) : t = . . . ,−1, 0, 1, ...\}$. Note that the regression equation (1) can be written, informally, as

$A(t) =(\quad B(t)$ with probability$(1 −\delta))$
$(A(t −1)$ with probability $\delta)$

Let A and B be the generic random variables for $A(t)$ and $B(t)$, respectively. The following are basic properties of a DAR(1) process.
• $\{A(t) : t = . . . ,−1, 0, 1, . . .\}$ is stationary.
• The stationary distribution of $\{A(t) : t = 0, 1, 2, . . .\}$ is the same as the distribution of B.
• The autocorrelation function $rA(k)$ for a DAR(1) process $\{A(t) : t = . . . ,−1, 0, 1, . . .\}$ is given by
as $rA(k) \equiv \mathrm{Cov}(A(t), A(t + k)) /\mathrm{Var}(A(t))= = \delta k,\ k= 0, 1, ...$
The model (1)is extended to higher orders as follows (see Jacob and Lewis :1978,1983)
The p-th order model, DAR(p) process $\{A(t) : t = . . . ,−1, 0, 1, . . .\}$, is given by$A(t) = (1− \alpha(t))A(t −\varphi(t)) + \alpha(t)B(t), ,.$

$A(t) = (1− \alpha(t))A(t −\varphi(t)) + \alpha(t)B(t),\ t = . . . ,−1, 0, 1, .$ *(2)*where $\{\alpha(t) : t = . . . ,−1, 0, 1, . . .\}$ and $\{B(t) : t = . . . ,−1, 0, 1, . \}$are as before and$\{\varphi(t) : t = . . . ,−1, 0, 1, . . .\}$ is a sequence of i.i.d. random variables taking values in theset$\{1, 2, . . p\}$ with$P(\varphi(t) = i) = \varphi i,\ i= 1, 2, . . . , p$.The processes $\{\alpha(t) : t = . . . ,−1, 0, 1, . . .\}$, $\{B(t) : t = . . . ,−1, 0, 1, . . .\}$ and $\{\varphi(t) : t =. ,−1, 0, 1, . . .\}$ are assumed to be independent. Note that the regression equation (2)can be written, informally, as
,

$$A(t)=\begin{cases} A(t-1) \text{ with probability } \delta\varphi 1, \\ B(t) \text{ with probability } 1-\delta, \\ A(t-2) \text{ with probability } \delta\varphi 2, \\ A(t-p) \text{ with probability } \delta\varphi p \end{cases} \quad (3)$$

It is seen that $\{A(t) : t = . . . ,−1, 0, 1, . . .\}$ is stationary and the stationary distribution o$\{A(t) : t = . . . ,−1, 0, 1, . . .\}$ is the same as the distribution of B. Furthermore, the autocorrelation function$r\ A(k)$ satisfies the following equations called Yule-Walker equations Jacobs and Lewis :
$rA(0) = 1, rA(k) =\delta \sum_{i=1}^{k} \varphi$
For some properties of DAR(p) process and related processes, see, for examples (Jacobs and Lewis ; McKenzie).
We consider the discrete time DAR(p)/D/2 queue, where the time is divided into slots of equal size and one slot is needed to serve a customer. The DAR(p) process $\{A(t),t = . . . ,−1, 0, 1, . . .\}$ serves as an arrival process, i.e., $A(t)$ represents the number of customers that arrive at the t th

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slot. The order of services are assumed to be based on the first come-

first-served policy. Furthermore, arriving customers during a single slot are served in random order. We assume the stability condition $\rho \equiv E[A(t)] < 1$

We consider a discrete-time queuing system where the time axis is segmented into a sequence of equal time intervals (called slots). It is assumed that all queuing activities (arrivals and departures)occur at the slot boundaries, and therefore they may occur at the same time. For mathematical clarity, we will suppose that the departures occur at the moment immediately before the slot boundaries and the arrivals occur at the moment immediately after the slot boundaries. Customers arrive according to a geometric arrival process with rate p, that is, p is the probability that an arrival occurs in a slot. If, upon arrival, the server is idle, the service of the arriving customer commences immediately. Otherwise, the arriving customer either with probability q^+ join the waiting line in order to be served (positive customer), or with complementary probability q^- the customer in service (negative customer) or all the customers in the system simultaneously(disaster). In the following two sections we will analyse both cases separately: negative customers and disasters.

$p_{1,0}$=s'p'+spq', $p_{2,0}$=s'pq' ,

It is always assumed that services can be started only at slot boundaries and their durations are

integral multiples of a slot duration. Service times are independent and geometrically distributed with probability $s^{/}$=1−s, where s is the probability that a customer does not finish his service in a slot.

We will suppose 0 < q+ < 1 and, in order to avoid trivial cases, 0 < p < 1 and 0 < s < 1. At time m+ (the instant immediately after time slot m), the system can be described by the process X_m , which denotes the number of customers in the sysytem (including the one in service if any).

It can be easily shown that {X_m, m € N} is the one dimensional Markov chain of our queuing system, whose states space is { 0,1,2,3…}.

Our first objective will be to 2nd the stationary distribution

$\pi_k = \lim_{n\to\infty} P[X_m = K]$, K ≥ 0

of the Marko chain {$X_{m,}$ m € N}. We introduce the auxiliary generating function

$\varphi(Z) = \sum_{k=1}^{\infty} \pi k\, Zk - 1$, $|$ z $|\leq 1$

in order to solve the Kolmogorov equations for the distribution πk. It should be pointed out that φ(z) is the marginal generating function of the number of customers in the waiting line when the server is busy.

3. The priority positive and negative queuing discipline

In this section we consider the (removal of customers from the head of the queue) discipline.

This is appropriate for modelling server breakdowns where a customer in service is lost.

The one-step transition probabilities $p_{k',k}$= P[X_{m+1}=k/X_{m+1}=k'] are given by the formulae

$P_{0,0}$=p', $p_{0,1}$=p, $p_{1,1}$=s'p' + spq', $p_{2,1}$=s'p' + spq' , $p_{3,1}$=s'pq' , $p_{k-1,k}$=s'pq' ,
$p_{k,k}$=s'pq + sp', $p_{k+1,k}$=s'p' + spq' , $p_{k+2,k}$=s'pq' where k$\geq$ 2 , and p'=1-p.

The Komolgorov equation for the distribution π_k are

π_0=p'π_0 + (s'p' + spq')π_1 + s'pq' π_2, (1)

π_1 = pπ_0 + (s'p + sp') π_1 + (s'p' + spq) π_2 + s'pq' π_3 , (2)

π_k = spq+ π_{k-1} + (s'pq+ + sp') π_k + (s'p' + spq') π_{k+1} + s'pq' π_{k+2} , K$\geq$ 2 (3)

and the normalization condition is $\sum_{k=0}^{\infty} \pi k = 1$.

Multiplying (3) by $z_k$and summing over k leads to

$[(s' + sz)\left(\frac{pq'}{z} + p'\right)$ + s'pq$^+$z –z]π_1 + [(s' + sz)pq' + s'p'z]π_2

+ s'pq'z π_3 = [(s' + sz)($\frac{pq'}{z}$+ p' + pq$^+$ z)– z]$\varphi(z)$. (4)

By substituting (1) (2) into (4) we have

[(s' + sz)($\frac{pq'}{z}$+ p' + pq$^+$ z)– z]$\varphi(z)$ = p(1-z)[π_0 + s'q' $\frac{z+1}{z}\pi_1$] ,

which can be written as

[spq$^+$ z^2 (p' + pq')z – pq']$\varphi(z)$ = -p(1-z)[$z\pi_0$ +s'q'(z + 1) π_1 , (5)

where _ = p= J s is the load of the system.

Note that the polynomial in the left-hand side of Eq. (5) has two roots z_{1*} and z_{2*} ,the conditions

2 , which satisfy the conditions -1 < z_{1*}< 0 and z_{2*}>0. Settings z=z_{1*} ineq (5), we get

$Z_{1*}\pi_0$ +s'q'(z_{1*} + 1) π_1 =0. (6)

Taking into account the expression

$\varphi(1) = \lim_{z\to 1} - \frac{p[z\pi 0 + s'q'(z+1)\pi 1]}{s\rho q + z2 - (p' + \rho q -)z - pq'}$ = (π_0 +2s'q$^-$ π_1)/(1-$\rho(q+ -q-)) * \rho$

And the normalization condition $\pi_0 + \varphi(1) = 1, we\ obtain$

(1+ 2 ρ q') π_0 + 2pq$^-$ π_1=1-ρ(q$^+$ -q$^-$) , (7)

The system of Eqs. (6)–(7) has a unique solution, since the determinant

$$\begin{vmatrix} z1* & s'q - (z1* + 1) \\ 1 + 2\rho q - & 2pq - \end{vmatrix}$$ =-q$^-$ [s'(z_{1*} + 1]) + +2p(q$^-$ - q$^+$)z_{1*})] (8)

is not equal to zero. This solution is given by

$^-\pi_0 = \frac{1-\rho(q+ - q-)}{(1-2pq+)z1* + 2\rho q- +1}(z_1^* + 1)$

$$\pi 1 = \frac{1-\rho(q+ \ -q-)}{(1-2pq+)z \ \ ^2* \ +2\rho q - \ +1}\frac{-z1*}{s'q -}$$

From the expressions for $\pi_0$and π_1, we conclude that ρ(q$^+$ - q$^-$)< 1 is a necessary condition for the ergodicity of the Markovchain.

Corollary1. The probability generating function of the number of customers in the waiting line (i.e., of the variable N) is given by

$\theta(z) = \pi_0 + \varphi(z) = \frac{1-p(q+ - q-)}{(1-2pq+)z1* + 2pq- +1}[z_1^* + 1 + \frac{1}{sq+(z2*-z)}]$

After a simple derivation exercise, we obtain

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$$\varphi k(z)=\frac{1-p(q+-q-)}{(1-2pq+)z1*+2pq-+1}\frac{k!}{sq+(z2*-z)k+1}\ , k\geq 0$$

Corollary2. (1) The steady-state distribution of the waiting line size is given by the following Formulae

$$P(N=0)=\pi_0+\pi 1=\frac{1-p(q+-q-)}{(1-2pq+)z1*+2pq-+1}\left(1+\frac{1-s'q'}{sq+z2*}\right)$$

$$P(N=k)=\pi_{k+1}=\frac{1-p(q+-q-)}{(1-2pq+)z1*+2pq-+1}\frac{1}{sq+(z2*)k+1}, k\geq 1$$

2) The steady-state distribution of the system size is given by the following formulae

$$P(S=0)=\pi_0=\frac{1-p(q+-q-)}{(1-2pq+)z1*+2pq-+1}(z_1*+1)$$

$$P(S=k)=\pi_k=\frac{1-p(q+-q-)}{(1-2pq+)z1*+2pq-+1}\frac{1}{sq+(z2*)k}, k\geq 1$$

4. Analysis of the DAR(P)/D/S

We consider the discrete time DAR(P)/D/Squeue where the time is divided into slots of equal size
one slot is needed to serve a packet by a server. Weassume that packet arrivals occur at the beginning of slots and departures occur at the end of slots. ADAR(P)/D/S, X (t): t =0; 1; 2; ..represents packet arrivalsso that X (t) is the number of packets arrivingat the beginning of the t^{th} slot.We analyze the DAR(P)/D/s queue. Let N(t) bethe number of packets in the system (we call it systamize) immediately before arrivals at the beginning of the tth slot. Then (N(t); X (t)): t =0; 1; 2; is a Markov process. Note that the Markov proces(N(t); X (t)): t =0; 1; 2; .. has M/G/1 . But, it is not easy to calculate the stationarydistribution of (N(t); X (t)): t =0; 1; 2; ..itself, because the number of phases is infinity. So, we find the stationary distribution of theMarkov process(N(t); X (t)): t =0; 1; 2; ..by introducing a new Markov process at the embedded epochs k : k =0; 1; 2; ..defined below.
Let $0=\tau 0<\tau 1<\tau 2<\tau 3 \ldots$ be the epochs defined by

$$\tau_k=\left\{\begin{array}{c}0\ ,\qquad k=0,\\ inf\{t>\tau k-1\ ;\ \alpha(t)=1\\ .\ or\ 0\leq X(t)\leq s-1\},\quad k=1,2,3\ldots.\end{array}\right\}$$

Let $N_k=N(\tau k)$, k=0; 1; 2; …..

$$j_k=\left\{\begin{array}{l}X(\tau k), if\ \alpha(\tau k)=0;\\ \quad s,\ if\ \alpha(\tau k)=1\end{array}\quad k=1,2,3\ldots..\right\}$$

Note that packet arrivals at and after τk are independent of the information prior to τk given J_k. For this, it is observed that N_k; J_k): k =0; 1; 2; …. is a Markov process with state space E =0; 1; 2; and that the Markov process N_k; J_k): k =0; 1; 2; …has the following transition probabilities.

1)For n=0; 1; 2; …and i=0; …s−1,

$$(n,i)\rightarrow\left\{\begin{array}{l}(max\{n-s+i,0\},i),\\ \ with\ probability\ \beta\\ (max\{n-s+i,0\},s)\\ with\ probability\ 1-\beta\end{array}\right.$$

Ai=

$$\begin{bmatrix}0 & .. & .. & 0 & .. & .. & 0\\ . & & & . & & & .\\ 0 & & & \beta & & & 1-\beta\\ . & & & . & & & .\\ . & & & . & & & .\\ 0 & & & hi\beta & & & hi(1-\beta)\end{bmatrix}$$

2)For n=0; 1; 2; : : :

$$;(n,i)=\left\{\begin{array}{c}(max\{n-s+i,0\},i), with\ probability\ bi\\ (n-s+i,s)\\ (0,s)\\ \\ (n+l,s) with\ probablity\end{array}\right.$$

where

$$\delta n0=\left\{\begin{array}{l}1\ if\ n=0,\\ 0\ if\ n>\geq 1,\end{array}\right.$$

$$Ai=\begin{bmatrix}0 & .. & .. & 0\\ .. & .. & .. & ..\\ .. & .. & .. & ...\\ 0 & .. & .. & gi-s\end{bmatrix}$$

$g_o=b_s$,

$$g_l=\frac{\sum_i bi+s(1-\beta)\beta\frac{l-1}{i}}{i}\quad ,l=1,2\ldots.$$

$$Bi=\sum_i Aj,\quad 1\leq i\leq s$$

We assume that the stability condition $E[X(t)]=\sum_{m=1}^{\infty}m(bm)<s$ is satisKed. Then, by matrix analytic methods in [8] Therefore, theMarkovprocess (N_k; J_k): k =0; 1; 2….. has the M/G/1 type structure (see [8]) of the one step transition probability matrix P:

the limiting probabilities of (N_k; J_k): k =0; 1; 2; : : :

$$\pi ni=\log_{k\to\infty}P\{Nk=n\ /k=i\}, n\geq 0, 0\leq i\leq s,$$

Are calculated as ,

1.set

$$P=\begin{bmatrix}Bs & As+1 & As+2 & .. & .. & & & \\ Bs-1 & As & As+1 & As+2 & .. & .. & & \\ .. & .. & .. & .. & & & & \\ .. & .. & .. & .. & & & & \\ B1 & A2 & A3 & A4 & A5 & .. & .. & \\ A0 & A1 & A2 & A3 & A4 & A5 & .. & .. \\ & A0 & A1 & A2 & A3 & A4 & A5 & .. & ..\\ & & A0 & A1 & A2 & A3 & A4 & A5 & ..\\ & & & .. & .. & .. & .. & .. & ..\\ & & & .. & .. & .. & .. & .. & ..\end{bmatrix}$$

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$$A'n=\begin{bmatrix} Asn & Asn+1 & .. & .. & As(n+1)-1 \\ Asn-1 & Asn & .. & .. & As(n+1)-2 \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ As(n+1)+1 & As(n+1)+2 & .. & \dots & Asn \end{bmatrix}$$

N= 0,1,2,3,………..

where A_l =O for l¡0. Set

$$B0'=\begin{bmatrix} Bs & As+1 & .. & .. & A2s-1 \\ As-1 & As & .. & .. & A2s-2 \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ B1 & A2 & .. & \dots & As \end{bmatrix},$$

B'n =A'n +1 , n=1,2,3,…..

2. Find the minimal nonnegative solution G of the matrix equation

$$G = \sum_{n=0}^{\infty} (A'n)Gn$$

For example, G is given by the iteration

G_0= 0, $G_{l+1} =\sum_{n=0}^{\infty} A'nG1n$, l=0,1,2………..

$$Pnj=\frac{\sum_{l=0}^{\infty}\sum_{i=0}^{S}\pi li\ E\ [\sum_{t=\tau k}^{\tau k+1-1}\{N(t)X(t)=(n,j)[(Nk,Jk)=(l,i)]}{\sum_{l=0}^{\infty}\sum_{i=0}^{S}\ \pi liE[\pi k+1-\pi k[(Nk,jk)=(l,i)]}.$$

Set

$$K= \sum_{n=0}^{\infty}(B'nGn)$$

3. Find a positive row vector k satisfying

kK= k

4. set

X_0=k ,

X_n= ($x_0 \sum_{i=0}^{\infty} B'n+1\,Gi + \sum_{l=1}^{n-1} xl\ \sum_{i=0}^{\infty} A'n-l+i+1\,Gi$). X(I - $\sum_{i=0(\,A'i+1\,G}^{\infty} i\,)^{-1}$, n=1,2,3….

Now we find the stationary distribution of the Markov process (N(t); X (t)): t = 0; 1; 2; : : :.Observe that ((N_k;J_k); τk): k = 0; 1; 2; : : : is aMarkov renewal sequence and that (N(t + τk); X (t + τk)): t = 0; 1; 2; : : :given (N(u); X (u)); τk ; (N_k; J_k) = (n; i)}is stochastically equivalent to (N(t); X (t): t =
0; 1; 2; : : : given(N_0; J_0) = (n; i). Hence(N(t);X (t)): t = 0; 1; 2; : : : is a discrete time Markov regenerative process with the Markov renewal sequence ((N_k; J_k); τk): k = 0; 1; 2; : : :.By Theorem 4 in Appendix A, the limiting probabilities (hence the stationary probabilities) p_{nj}= $\lim_{t\to\infty}$P(N(t); X (t)) = (n; j); n;j=0; 1; 2; : : :, of (N(t); X (t)): t = 0; 1; 2; : : are given by

Observe that

$$E[\sum_{t=\tau k}^{\tau k+1\ -1} 1\{\,N(t),X(t)=(n,j)\}|\,(Nk,Jk)=(l,i)\ =\begin{cases} 1\ if\ i=j\,,0\le i\le s-1\ and\ n=l, \\ bj\ \ if\ i=s\,,0\le j\le s-1\ and\ n=l, \\ \frac{bs}{1-\beta}\ \ if\ i=s,j=s\ ansd\ n=l\,, \\ bj\ \beta\frac{n-l}{j-s}\ if\ i=s\,,j>s\,,n\ge l\ and\ j-s\ divides\ n-l, \\ 0\ otherwise \end{cases}$$

Therefore the numerator of the right hand side of (3) is

$$\begin{cases} \pi nj+\ \pi nsbj,\ \ ,0\le j\le s-1\,, \\ \frac{\pi nsbs}{1-\beta}\,,j=s, \\ \sum_{i=0}^{\frac{n}{j-s}}\pi n-i(j-s),sbj\beta i,\ \ j\ge s+1 \end{cases}$$

Observe that

$$E[\tau_{k+1}-\tau_k|(N_k,J_k)=(l,i)] =\begin{cases} 1\ \ if\ \ ,0\le j\le s-1\,, \\ \sum_{s=0}^{s-1} bs+\ \sum_{s=c}^{\infty} bs\frac{1}{1-\beta}\ \ \ if\ i=s. \end{cases}$$

Therefore the denominator of the right hand side of (3) is

$$\sum_{l=0}^{\infty}.\sum_{i=0}^{s-1}\pi li+\ \ \ \sum_{n=0}^{\infty}\pi ls\left(\sum_{l=0}^{\infty}bs+\sum_{i=0}^{\infty}bs\ \frac{1}{1-\beta}\right) \qquad (4)$$

$$P(W=w) = \frac{\text{Mean number of arrivals in a slot at steady state whose waiting time is w}}{\text{Mean number of arrivals in as lot}} \qquad (6)$$

{J_k:k =0; 1; 2; : : :} whose transition probability matrik is

P((Jk+1=j|Jk))0≤ i , $j \le s$

$$\begin{bmatrix} \beta & .. & 0 & 0 & 1-\beta \\ .. & & .. & .. & \\ 0 & & \beta & 0 & 1-\beta \\ .. & & .. & .. & .. \\ 0 & & 0 & \beta & 1-\beta \\ \beta b0 & & \beta b1 & \beta bs\_1 & 1-\beta\sum_{s=c0}^{s-1} bs \end{bmatrix}$$

By solving the balance equations for the stationarydistribution of the Markov process J_k:k =0; 1; 2; …..
we obtain

$$\sum_{l=0}^{\infty}\pi li =\begin{cases} \frac{\beta bi}{1-\beta\sum_{s=c}^{\infty} bs}\ \ \ ,0\le i\le s-1\,, \\ \frac{1-\beta}{1-\beta\sum_{i=s}^{\infty} bi}\,,i=s \end{cases} \qquad (5)$$

By substituting (5) into (4), we obtain the denominator of the right hand side of (3) as $\frac{1-\beta}{1-\beta\sum_{t=s}^{\infty} bt}$,

Thus, we have the following theorem.

Theorem1. The limiting probabilities (hence the stationary probabilities)

Pnj=$\lim_{t\to\infty} P$\{ N(t),X(t)=(n,j)\}, n,j=0,1,2,….., of {(N(t),X(t)) : t=0,1,2,….} are given by

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p_{nj} =

$$\begin{cases} \mu-1(\pi nj+\pi nbj) & ,0\le j\le s-1, \\ \mu-1\frac{\pi nsbs}{1-\beta}, j=s, & \\ \mu-1\sum_{i=0}^{\frac{n}{j-s}}\pi n-i(j-s), sbj\beta i, & j\ge s+1 \end{cases}$$

where $(\mu)-1=1-\beta\sum_{t=s.}^{\infty} \quad bt$

Now we find the stationary distribution of the waiting time of a packet. Let W denote the waiting
time of an arbitrary packet at steady state. Then for w=0 ,1,2…,

Suppose that there are n packets immediately before arrivals at the beginning of the tth slot and that the number of packet arrivals is j at the beginning of the tth slot, i.e., N(t)=n and X (t)=j. Then the number of packets whose waiting time is w among the ones who arrive at the beginning of the tth slot is

$$\begin{cases} min\{s(w+1)-n,j\}, & sw<n<s(w+1) \\ min\{n+j-sw,w\}, & n\le sw<n+j \\ 0, & otherwise \end{cases}$$

Therefore the mean number of arrivals in a slot at steady state whose waiting time is w is

$$\sum_{n=0}^{sw}\cdot\sum_{j=sw-n+1}^{\infty} pnjmin(n+j-sw,s)+$$
$$n=sw+1sw+1-1j=1\infty i=0\infty pnj \min\{sw+1-n,j\}$$

Since the mean number of arrivals in a slot is at the following theorem is obtained from (6).

Theorem2. The distribution of the waiting time Wof an arbitrary packet is given by

P(W=w) =

$$\frac{1}{\lambda}\sum_{n=0}^{sw}\cdot\sum_{j=sw-n+1}^{\infty} pnjmin(n+j-sw,s)+$$
$$n=sw+1sw+1-1j=1\infty i=0\infty pnj \min\{sw+1n,j\},$$

W= 0,1,2……..

5. Conclusions:

In this paper we consider a discrete time dual server queuing system, queuing networks with negative customers, signals, triggers where they arrive Gaussian process of order p (DAR(P)/D/s, and the service time of a customer is one slot. In contrast with the normal positive customers, negative customers arriving to a non-empty queue remove and work from the queue For this queuing system, we give an expression for the mean queue size. Further we suggest that approximation methods for the mean queue size which is based on matrix method is more suitable than the generating method. .

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