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System Order Reduction of Transfer Function via Haar Functions in Time Domain

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Abstract

This paper presents a system order reduction method of transfer function using Haar functions based on the approximation and Haar transforms method. A high order system often contains less significant poles that have little effect on the system response. Therefore, if possible it is useful and desirable to find a low-order approximating system from the original high-order system, so that the handling effort such as system analysis and design can be reduced. Walsh functions were completed from the incomplete orthogonal function of Rademacher in 1923. And also Haar function set forms a complete set of orthogonal rectangular functions similar in several respects of the Walsh functions. The method adopted in this paper is that of system approximation using Haar transform. This approach provides a more efficient and convenient method for the system order reduction.

Keyword: transfer function, Haar functions, transform, system order reduction

I. Haar Functions

For analysis and design purposes, sort out the poles that have a dominant effect on the transient response is important. On the other hand, the poles that are far away from the imaginary axis in the left half s-plane or relative to the dominant poles are insignificant. In system design, we can use dominant poles of transfer function to control dynamic performance of the system, but less important or insignificant poles can be neglected with regard to the transient response. Thus, now we can consider neglecting of insignificant poles of transfer function and reducing the system order. Haar functions that are introduced in this paper are useful to approximation of transfer function for system analysis and reduction. The Haar functions were established rather earlier than the Walsh functions by the Hungarian mathematician. Alfred Haar described a set of orthogonal functions, each taking essentially only two values and providing a simple convergence and expansion of system. The Haar functions form an orthogonal and orthonormal system of periodic square waves. If we consider the time base to be defined as $0 \le t \le 1$ then, the Haar functions is described as follows. And its waveform of first eight Haar functions is shown in figure 1.

$$h(0,t) = 1 \quad for \ 0 \le t \le 1$$

$$h(1,t) = \begin{cases} 1 & for \ 0 \le t \le \frac{1}{2} \\ -1 & for \ \frac{1}{2} \le t \le 1 \end{cases} \tag{1.1}$$

$$h(2,t) = \begin{cases} \sqrt{2} & for \ 0 \le t \le \frac{1}{4} \\ -\sqrt{2} & for \ \frac{1}{4} \le t \le \frac{1}{2} \\ 0 & for \ \frac{1}{2} \le t \le 1 \end{cases} \tag{1.2}$$

$$h(2^p+n,t) = \begin{cases} \sqrt{2^p} & for \ \frac{n}{2^p} \le t \le \frac{n+\frac{1}{2}}{2^p} \\ -\sqrt{2^p} & for \ (n+\frac{1}{2})/2^p \le t \le (n+1)^{2^p} \\ 0 & for \ elsewhere \end{cases} \tag{1.3}$$

where $p=0, 1, 2....$ $n=0, 1, ...2^p-1$

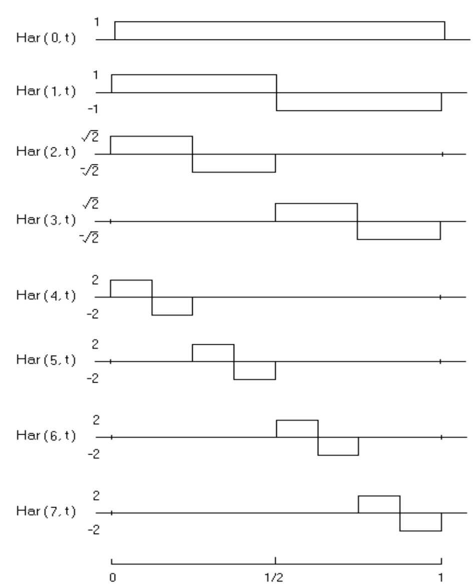

Figure 1. The first eight Haar functions

From the definition in equation (1.1), it can be seen that

Haar functions are orthogonal, thus we can obtain (1.2).

$$\int_0^1 h(m,t)h(n,t) = \begin{cases} 1 & for\ m = n \\ 0 & for\ m \neq n \end{cases} \qquad (1.2)$$

For instance, the first four Haar functions H_4 can be written as equation (1.3).

$$H_4 = \begin{bmatrix} H_0(t) \\ H_1(t) \\ H_2(t) \\ H_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix} \qquad (1.3)$$

II. Haar Transform

A function $f(t)$ is absolutely integrable on then it can be expanded as an infinite series in term of Haar functions.

$$f(t) = f_0 h_0(t) + f_1 h_1(t) + f_2 h_2(t) + \cdots$$
$$= \sum_{i=0}^{\infty} f_i h_i(t) \qquad (2.1)$$

Where f_i is the ith sequentially ordered coefficient of the Haar functions expansion of function $f(t)$ and h_i is the ith ordered Haar functions. The coefficient of the Haar functions expansion is given as equation (2.2).

$$f_i = \int_0^1 f(t)h_i(t)dt \qquad (2.2)$$

We can get the approximation of $f(t)$ using Haar transform and its matrix expression.

$$f(t) = \sum_{i=0}^{n-1} f_i h_i(t) = F_n^T H_n(t) \qquad (2.3)$$

where F_n is coefficient vector of $f(t)$ and $H_n(t)$ is its Haar functions vector. T denotes transposition. For example, if $f(t)=t$, $f(t)$ can be transformed and approximated using Haar functions and the result is shown in figure 2.

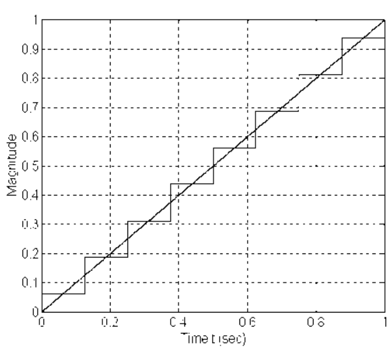

Figure 2. Haar transform of $f(t)=t$ with $n=8$

III. System Order Reduction using Haar Functions

Given a high-order transfer function $M_H(s)$, we can find a low-order transfer function $M_L(s)$ as an approximation. A method of approximating high-order system by low-order system is based on one in the sense that the frequency responses of two systems are similar. Let the high-order system transfer function be written by equation (3.1) and the transfer function of the approximating low-order system be represented by equation (3.2).

$$M_H(s) = K\frac{1+b_1 s+b_2 s^2+\cdots+b_m s^m}{1+a_1 s+a_2 s^2+\cdots+a_n s^n} \qquad (3.1)$$

$$M_L(s) = K\frac{1+c_1 s+c_2 s^2+\cdots+c_q s^q}{1+d_1+d_2 s^2+\cdots+d_p s^p} \qquad (3.2)$$

where $n \geq m$, $n \geq p \geq q$. And $s=jw$ is applied to above equations, thus we can obtain equation (3.3) and (3.4) respectively.

$$M_H(jw) = k\frac{A(w)+jw\underline{A}(w)}{B(w)+jw\underline{B}(w)} \qquad (3.3)$$

$$M_L(jw) = k\frac{C(w)+jw\underline{C}(w)}{D(w)+jw\underline{D}(w)} \qquad (3.4)$$

In this case, the zero frequency gain K of the two transfer functions is the same. Thus, we can obtain the criterion of finding the low-order $M_L(s)$, given $M_H(s)$, is that the following relation should be satisfied (3.5).

$$\frac{|M_H(jw)|^2}{|M_L(jw)|^2} = 1 \qquad (3.5)$$

Equating both side of equation (3.5) and satisfying the condition, similar relationships can be obtained for coefficients of equation (3.3) and (3.4). Now we can apply Haar functions and its transform to determine the coefficients of equation (3.4) as follows.

$$\mathrm{C}(w) = \sum_{i=0}^{n-1} c_i h_i(w), c_i = \int_0^1 A(w)h_i(t)dt \qquad (3.6)$$

$$\underline{C}(w) = \sum_{i=0}^{n-1} \underline{c_i} h_i(w), \underline{c_i} = \int_0^1 \underline{A}(w)h_i(t)dt \qquad (3.7)$$

$$D(w) = \sum_{i=0}^{n-1} d_i h_i(w), d_i = \int_0^1 B(w)h_i(t)dt \qquad (3.8)$$

$$\underline{D}(w) = \sum_{i=0}^{n-1} \underline{d_i} h_i(w), \underline{d_i} = \int_0^1 \underline{B}(w)h_i(t)dt \qquad (3.9)$$

Applying Haar functions and similar relationships, we can define the coefficients from (3.6) to (3.9) conveniently. This

method is useful to convert high-order transfer function into low-order.

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IV. Examples

Consider that the forward path transfer function of unity feedback control system is given as figure 3,

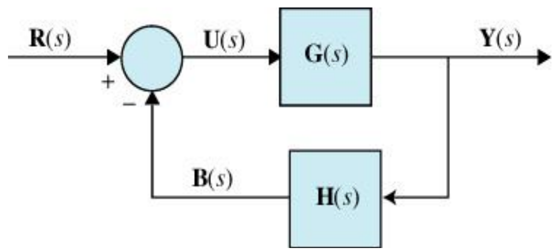

Figure 3. Block diagram of feedback control system

$$G(s) = \frac{8}{s(s^2+6s+12)},\ H(s) = 1 \tag{4.1}$$

Thus, the system transfer function is written as,

$$M_H(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{8}{s^3+s^2+12s+8}$$

$$= \frac{1}{1+1.5s+0.75s^2+0.125s^3} \tag{4.2}$$

From equation (4.2), simplified low-order system $M_L(s)$ is the second order system and the transfer function is determined as follow. In order to compare with the proposed method in this paper, the result (4.3) can be obtained using one of the numerical methods.

$$M_L(s) = \frac{1}{1+m_1 s+m_2 s^2}$$

$$= \frac{1}{1+1.171s+0.433s^2}$$

$$= \frac{2.31}{s^2+2.7045s+2.31} \tag{4.3}$$

Now we apply the Haar functions and its transform that is suggested in this paper to get the low-order transfer function $M_L(s)$ from the original high-order transfer function M_H(s). Using equation (3.3) and (3.4), we can get (4.4) and (4.5).

$$M_H(jw) = \frac{1}{1+1.5jw+0.75(jw)^2+0.125(jw)^3}$$

$$= \frac{1}{(1-0.75w^2)+jw(1.5-0.125w^2)} \tag{4.4}$$

$$M_L(jw) = \frac{1}{1+m_1 jw+m_2(jw)^2}$$

$$= \frac{1}{(1-m_2w^2)+jwm_1} \tag{4.5}$$

From similar relationships of equation (3.5), d_1 and d_2 are represented as follows.

$$m_1=1.5-0.125w^2,\ m_2=0.75 \tag{4.6}$$

And also, equation (4.7) can be obtained by equation (3.4) and (4.6).

$$\mathrm{C}(\mathrm{w}) = 1, \underline{C}(w) = 0$$

$$D(w) = 0.75, \underline{D}(w) = 1.5-0.125w^2 \tag{4.7}$$

In order to determine the coefficients of $\underline{D}(w)$, we can apply Haar functions and transform to $\underline{D}(w)$, then equation (4.7) is represented by equation (4.8).

$$\underline{d_i} = \int_0^1 h_i(w)(1.5-0.125w^2)dw \tag{4.8}$$

$$\begin{aligned} d_0 &= 1.4583 \\ d_1 &= 0.0313 \\ d_2 &= 0.0055 \\ d_3 &= 0.0165 \\ d_4 &= 0.0009 \\ d_5 &= 0.0029 \\ d_6 &= 0.0050 \\ d_7 &= 0.0069 \end{aligned} \tag{4.9}$$

where i=1, 2, 3, …7.(n=8). Continuously we can define the discrete values of coefficients d_i using Haar operational matrix and equation (4.9).

$$d_i^* = H_8 x d_i$$

where d_i^* denotes discrete values of d_i and H_8 denotes Haar operational matrix with n=8.

$$\begin{aligned} d_0^* &= 1.5723 \\ d_1^* &= 1.5409 \\ d_2^* &= 2.0119 \\ d_3^* &= -0.0115 \\ d_4^* &= 2.8540 \\ d_5^* &= -0.1120 \\ d_6^* &= -0.0040 \\ d_7^* &= -0.0038 \end{aligned} \tag{4.10}$$

The results in time domain are shown in figure 4 to 6. Transfer function(TF) analysis between the original third-order system and the numerical second-order system is shown in figure 4. The dashed graph shows the original third-order system and the dotted graph displays the numerical second-order system. Figure 5 shows analysis between the original third-order system and the proposed second-order system. In the figure, solid line stands for the proposed second-order system. And three transfer functions in time domain analysis is shown in figure 6.

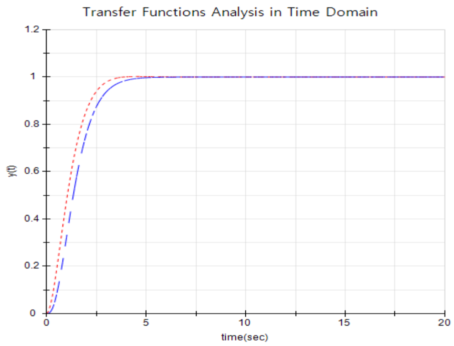

Figure 4. Original and numerical TFs analysis

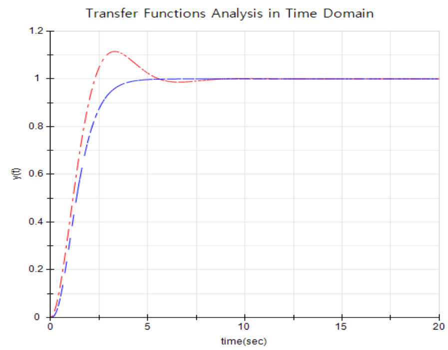

Figure 5. Original and proposed TFs analysis

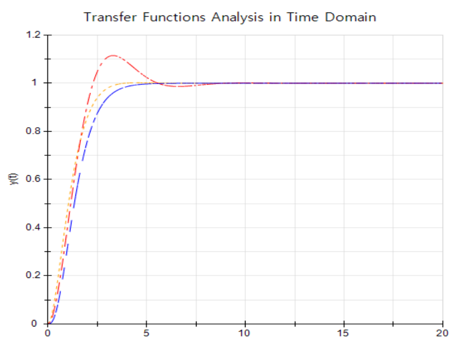

Figure 6. Transfer functions analysis

IV. Conclusions

We have seen that the Haar functions and its transforms can be used as an approximating set of system order reduction of transfer functions. System order reduction is useful for system analysis and design because insignificant pole can be neglected with regard to the transient response. The proposed transfer function has the overshoot greater than the original third-order system. The cause of this phenomenon is based on an error by approximation. Nevertheless proposed method for approximating transfer function is simple and useful.

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