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SOLVING LARGE-SCALE THREE-LEVEL LINEAR FRACTIONAL PROGRAMMING PROBLEM WITH ROUGH COEFFICIENT IN CONSTRAINTS

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Abstract : Large-Scale three level fractional problem is considered in this paper with a rough parameter in Constraints, in order to solve this problem, the intervals technique is used to convert rough parameters in constraints into equivalent crisp, Then Tailor Series transformation is used to solve the fractional problem, then a proposed model has been constructed to solve the decision conflict of three-level problem, finally a Decomposition Technique is used to solve Large-Scale Problem. An auxiliary problem is discussed as well as an example is presented.

Keywords

Three-level problem; Fractional Problem; Rough interval Coefficient; Large-Scale; Tailor Series , fuzzy, decomposition.

Introduction

Hierarchical optimization or multiple level programming (MLP) techniques are formulated in order to solve decentralized planning problems involving several decision makers (DMs) in a hierarchical organization based on the concept of Stackelberg game theory [1].

A three level programming concept is about that the first level decision maker (FLDM) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the second level decision maker (SLDM) is then submitted and modified by the FLDM; Finally, the third level decision maker (TLDM) is submitted and modified by the FLDM and SLDM with consideration of the overall benefit for the organization; and the process continued until an optimal solution is reached

During the past few decades, many methodological developments have been reported for multi-level programming problem (MLPP). However, these methods are proven to be computationally ineffective and can handle only simple (MLLPs). To overcome the shortcomings of the traditional methods, the concept of membership function of fuzzy set theory was incorporated for large and complex hierarchical optimization problems. Lai [1] at first proposed a new solution concept based on tolerance membership functions as well as multiple objective Optimizations to develop an effective fuzzy approach for solving (MLPP).

Shih et al. [2] extended Lai's concept and proposed a supervised search procedure by employing non-compensatory max-min aggregation operator for solving MLPP.

Tirayaki [3] discussed interactive compensatory fuzzy programming for decentralized MLLPs to obtain a preferred compensatory compromise Pareto-optimal solution.

Pramanik and Roy [4] developed a fuzzy goal programming (FGP) technique for MLPPs for proper distribution of decision powers to the DMs to reach a satisfying decision.

Baky [5] presented two FGP algorithms to solve multi-objective MLPPs to achieve highest degree of each of the membership goals by minimizing over and under deviational variables. Arbaiy and Watada [5] discussed additive FGP model for solving multi-objective MLPPs for obtaining satisfaction solution.

Recently, notable studies have been done in the area of large scale linear and nonlinear programming problems with block angular structure

Dantzig and Wolfe decomposition method [10] has provided the foundation for much of the research in the area of the large scale programming problems in which breaking the decision space among several planning subunits. These sub-units interact through a set of corporate constraints involving the decision variables of all the divisions. The remaining constraints can be apportioned to each subunit, with each constraint including only the decision variable of a single subunit.

When the objective functions of level DMs of a MLPP are linear fractional in nature, then the MLPP is called multi-level linear fractional programming problem (MLFPP)[14]. Lachhwani and Poonia[6] proposed a different FGP[13] approach for MLFPP by defining separate membership functions for numerator and denominator functions of the fractional objective functions at each level.

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Problem Formulation and Solution Concept

Consider a three-level Large-Scale Fractional programming problem of maximization-type with random rough coefficient in the constraint scan be written as:

[1^{st}Level]

$$\underset{x_1}{\text{Max}}\, F_1 = \frac{a_1 x + \alpha_1}{b_1 x + \beta_1}, (1)$$

where $x_2, \ldots, x_m$ solves

[2^{nd}Level]

$$\underset{x_2}{\text{Max}}\, F_2 = \frac{a_2 x + \alpha_2}{b_2 x + \beta_2}, (2)$$

where $x_3, \ldots, x_m$solves

[3^{rd}Level]

$$\underset{x_3}{\text{Max}}\, F_3 = \frac{a_3 x + \alpha_3}{b_3 x + \beta_3}, (3)$$

where$x_4, \ldots, x_m$solves

Subject to

$$G = \{[[a_0^2, a_0^3], [a_0^1, a_0^4]]x \le [[b_0^2, b_0^3], [b_o^1, b_0^4]], (4)$$
$$[[d_1^2, d_1^3], [d_1^1, d_1^4]]x_1 \le [[b_1^2, b_1^3], [b_1^1, b_1^4]],$$
$$[[d_2^2, d_2^3], [d_2^1, d_2^4]]x_2 \le [[b_2^2, b_2^3], [b_2^1, b_2^4]],$$
$$\vdots$$
$$[[d_m^2, d_m^3], [d_m^1, d_m^4]]x_m \le [[b_m^2, b_m^3], [b_m^1, b_m^4]],$$
$$x_m \ge 0 \}.$$

In the above Problem (1) - (4), $F_k : R^m \to R, (k = 1,2,3)$ be the first level, the second level, and the third level objective function, respectively. Moreover, the FLDM has x_1 indicating the first decision level integer choice, the SLDM and the TLDM have x_2 and x_3 indicating the second and the third decision level choice, respectively.

Taylor's Series Approach for Linearization of Fractional Objectives Functions [6]

We can transform the objective functions by using 1^{st} order Taylor series [6] into linear functions.

Taylor's approach can be summarized in the following steps:-

Step1: Determine $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*)$ which is the value that is used to maximize the i^{th}objective functions$F_i(x)$, (*i* = *1,2,..., m*) where *n* is a number of the variables.

Step2: Transform the objective functions $F_i(x)$(*i* = *1,2,..., m*) by using the 1^{st}order Taylor polynomial series in the following form stated as:

$$F_i(x) \cong F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_i^*)}{\partial x_j}, (i = 1,2,\ldots,m)$$

Basic Preliminaries about RI

Conversion of Linear Programming problem with rough interval coefficients into upper and lower approximation is usually hard work for many cases, but transformation process needs the following definitions to be known:

Definition 1 [7]

RI can be considered as a qualitative value from vague concept defined on a variable x in R.

Definition 2 [7]

The qualitative value A is called a rough interval when one can assign two closed intervals A_* and A^* on R to it where $A_* \subseteq A^*$.

Definition 3 [7]

A_* and $A^*$are called the Lower Approximation Interval (LAI) and the Upper Approximation Interval (UAI) of *A*, respectively. Further, *A* is denoted by $A = (A_*$ and $A^*)$.

Definition 4 [7]

Consider all of the corresponding Linear Programming with Interval Coefficients (LPIC) and LPof Problem (1)-(4):

1. The interval $[f_*^L, f_*^U]([f^{*L}, f^{*U}])$ is called the surely (possibly) optimal range of Problem (1) - (4) if the optimal range of each LPIC is a superset (subset) of$[f_*^L, f_*^U]([f^{*L}, f^{*U}])$.
2. Let $[f_*^L, f_*^U]([f^{*L}, f^{*U}])$ be surely(possibly) an optimal range of Problem (1)-(4), then the rough interval

$[[f_*^L, f_*^U][f^{*L}, f^{*U}]]$ is called the rough optimal range of Problem (1)-(4).

3. The optimal solution of each corresponding LPIC of Problem (1)-(4) which its optimal value belongs to $[f_*^L, f_*^U]([f^{*L}, f^{*U}])$ is called a completely(rather) satisfactory solution of Problem (1) -(4).

Now, the equivalent problem of the FLDM, SLDM, and TLDM using interval method [2] can be obtained by getting the surely optimal range of Problems (1) - (4), which resulted in the following two large scale linear programming problems.

LALB	LAUB
$f_K^2 = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}, k$ subject to $\sum_{j=1}^{m}\sum_{i=1}^{n} a_0^3 x \le b_0^2,$ $\sum_{i=1}^{n} d_1^3 x_1 \le$ $\sum_{i=1}^{n} d_2^3 x_2 \le$ $\sum_{i=1}^{n} d_m^3 x_m \le$ $x_m \ge 0.$	$f_K^3 = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}, \quad k$ subject to $\sum_{j=1}^{m}\sum_{i=1}^{n} a_0^2 x \le b_0^3,$ $\sum_{i=1}^{n} d_1^2 x_1 \le b_1^3,$ $\sum_{i=1}^{n} d_2^2 x_2 \le b_2^3,$ $\sum_{i=1}^{n} d_m^2 x_m \le b_m^3,$ $x_m \ge 0.$

Table (1): Lower Approximations of Rough Intervals Coefficients of the FLDM, SLDM, and TLDM

While the possibly optimal range of the FLDM, SLDM, and TLDM using interval method [2] can be obtained by getting the possibly optimal range of Problems (1) - (4), which resulted in the following two large scale linear programming problems.

UALB	UAUB
$f_k^1 = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1},$ subject to $\sum_{j=1}^{m}\sum_{i=1}^{n} a_0^4 x \le b_0^1,$ $\sum_{i=1}^{n} d_1^4 x_1 \le b_1^1,$ $\sum_{i=1}^{n} d_2^4 x_2 \le b_2^1,$ $\sum_{i=1}^{n} d_m^4 x_m \le b_m^1,$ $x_m \ge 0.$	$f_k^4 = \dfrac{a_1 x + \alpha_1}{b_1 x + \beta_1}, \quad k = 1,2,3, \quad (8)$ subject to $\sum_{j=1}^{m}\sum_{i=1}^{n} a_0^1 x \le b_0^4,$ $\sum_{i=1}^{n} d_1^1 x_1 \le b_1^4,$ $\sum_{i=1}^{n} d_2^1 x_2 \le b_2^4,$ $\sum_{i=1}^{n} d_m^1 x_m \le b_m^4$ $x_m \ge 0.$

Table (2): Upper Approximations of Rough Intervals Coefficients of The FLDM, SLDM, and TLDM

So, the problem of three-level Large-Scale Linear programming problem with rough interval parameters in the constraints (1) - (4) can be converted into twelve large scale linear programming problems.

A Decomposition Algorithm for Three Levels Large Scale Linear Programming Problem

To solve the three levels large scale linear integer programming problem based on the decomposition algorithm [3] and constraint method [4]. The FLDM gets the optimal solution using decomposition algorithm [3] by breaking the large scale problem into n-sub problems that can be solved directly. Then by inserting the FLDM decision variable to the SLDM for him/her to seek the optimal solution using decomposition method [8]. Finally, the TLDM does the same action till he/she obtains the optimal solution of his problem.

The suggested algorithm can be summarized in the following manner:

Step 1. The FLDM formulate his problem.

Step2. If the FLDM obtain his optimal solution set $(x_1, x_2) = (x_1^F, x_2^F)$ go to Step 9 , otherwise go to step 3.

Step 3. Compute $\Re(\tilde{A}) = a + b + \frac{1}{2}(d - c)$ where $\tilde{A} = (a,b,c,d) \in F(R)$, go to Step 4.

Step 4. Set $k = 1$.

Step 5. Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk}$, go to Step 6.

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Step 6. If $z^*_{jk} - c^*_{jk} \le 0$, then go to Step 7; otherwise, the optimal solution has been reached, go to Step8.

Step 7. Set $k = k+1$, go to Step 5.

Step 8. The decision maker use interval method to transform the problem with rough parameters into crisp model.

Step 9. If SLDM obtain his optimal solution set $(x_1, x_2, x_3, x_4) = (x_1^F, x_2^F, x_3^S, x_4^S)$ then go to Step 11 , otherwise go to step 10.

Step 10. The SLDM formulate his problem, go to Step 3.

Step 11. If the TLDM obtain the optimal solution go to Step 13 , otherwise go to Step 12.

Step 12. The TLDM formulate his problem, go to Step 3.

Step 13. $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_{6,}^T ..., x_m^T)$ is an optimal solution for three-level large scale linear programming problem with rough parameters problem, then stop .

Remark1. The lingo package is suggested as a basic solution tool.

Numerical Example

[1st Level]

$$\max_{x_1,x_2} f_1(x) = \max_{x_1,x_2} \frac{15x_1 + 6x_2 + 9x_5 + 4x_6}{x_1 + 4x_2 + 6}$$

where x_3, x_4, x_5, x_6 solve

[2nd Level]

$$\max_{x_3,x_4} f_2(x) = \max_{x_3,x_4} \frac{6x_3 + 4x_4 + 4x_5 + x_6}{6x_3 + 4x_4 + 2}$$

where x_5, x_6 solve

[3rd Level]

$$\max_{x_5,x_6} f_3(x) = \max_{x_5,x_6} \frac{x_1 + 4x_5 + 9x_6}{x_5 + x_6 + 6}$$

Subject to

$$\{[[1,2],[1,3]]x_1 + [[2,3],[1,4]]x_2 + [[1,3],[1,4]]x_3 + [[1,3],[1,5]]x_4 + [[2,3],[1,5]]x_5 + [[1,2],[1,4]]x_6 \le [[30,40],[25,50]],$$

$$[[2,3],[1,5]]x_1 + [[1,2],[1,3]]x_2 \le [[8,15],[5,20]],$$
$$[[1,3],[1,5]]x_3 + [[1,2],[1,4]]x_4 \le [[8,10],[5,12]],$$
$$[[1,2],[1,3]]x_5 + [[2,3],[1,4]]x_6 \le [[5,15],[4,18]],$$
$$[x_1],[x_2],[x_3],[x_4],[x_5],[x_6] \ge 0.$$

FLDM problem using Taylor's series and decomposition algorithm

The equivalent problem of the FLDM with rough coefficients in objective function by using interval method [11] can be written as:-

Lower Approximations of RI Coefficients of The FLDM	
(LALB)^F	**(LAUB)^F**
$f_1^2 = \max \frac{15x_1 + 6x_2 + 9x_5 + 4x_6}{x_1 + 4x_2 + 6}$, Subject to $2x_1 + 3x_2 + 3x_3 + 3x_4 + 3x_5 + 2x_6 \le 30,$ $3x_1 + 2x_2 \le 8,$ $3x_3 + 2x_4 \le 8,$ $2x_5 + 3x_6 \le 5,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$	$f_1^3 = \max \frac{15x_1 + 6x_2 + 9x_5 + 4x_6}{x_1 + 4x_2 + 6}$, Subject to $x_1 + 2x_2 + x_3 + x_4 + 2x_5 + x_6 \le 40,$ $2x_1 + x_2 \le 15,$ $x_3 + x_4 \le 10,$ $x_5 + 2x_6 \le 15,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$
Upper Approximations of RI Coefficients of The FLDM	
(UALB)^F	**(UAUB)^F**
$f_1^1 = \max \frac{15x_1 + 6x_2 + 9x_5 + 4x_6}{x_1 + 4x_2 + 6}$, Subject to $3x_1 + 4x_2 + 4x_3 + 5x_4 + 5x_5 + 4x_6 \le 25,$ $5x_1 + 3x_2 \le 5,$ $5x_3 + 4x_4 \le 5,$ $3x_5 + 4x_6 \le 4,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$	$f_1^4 = \max \frac{15x_1 + 6x_2 + 9x_5 + 4x_6}{x_1 + 4x_2 + 6}$, Subject to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,$ $x_1 + x_2 \le 20,$ $x_3 + x_4 \le 12,$ $x_5 + x_6 \le 18,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$

Table (1): Lower and Upper Approximations of RICoefficients of the FLDM

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Then, the objective functions of the FLDM in Table (2) are transformed by using 1^{st} order Taylor polynomial series to linear functions as follows:

Lower Approximations of RI Coefficients of The FLDM	
$(LALB)^F$	**$(LAUB)^F$**
$f_1^{\,2} = \max 1.08x_1 - 0.$ $+0.36x_6 + 1.41,$ Subject to $2x_1 + 3x_2 + 3x_3 + 3x_4$ $+3x_5 + 2x_6 \le 30,$ $3x_1 + 2x_2 \le 8,$ $3x_3 + 2x_4 \le 8,$ $2x_5 + 3x_6 \le 5,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge$	$f_1^{\,3} =$ $\max 1.08x_1 - 0.57x_2 + 0.8$ $+0.36x_6 + 1.41,$ Subject to $x_1 + 2x_2 + x_3 + x_4$ $+2x_5 + x_6 \le 40,$ $2x_1 + x_2 \le 15,$ $x_3 + x_4 \le 10,$ $x_5 + 2x_6 \le 15,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$
Upper Approximations of RI Coefficients of The FLDM	
$(UALB)^F$	**$(UAUB)^F$**
$f_1^{\,1} =$ $\max 1.08x_1 - 0.57x_2$ $+0.36x_6 + 1.41,$ Subject to $3x_1 + 4x_2 + 4x_3 + 5x_4$ $5x_5 + 4x_6 \le 25,$ $5x_1 + 3x_2 \le 5,$ $5x_3 + 4x_4 \le 5,$ $3x_5 + 4x_6 \le 4,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge$	$f_1^{\,4} =$ $\max 1.08x_1 - 0.57x_2 + 0.81x_5$ $+0.36x_6 + 1.41,$ Subject to $x_1 + x_2 + x_3 + x_4 +$ $x_5 + x_6 \le 50,$ $x_1 + x_2 \le 20,$ $x_3 + x_4 \le 12,$ $x_5 + x_6 \le 18,$ $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$

Table (2): Transformation of the FLDM Objective Functions to Linear Functions

After that, apply the decomposition algorithm on the FLDM to solve LSLP problems in Table (2) and get the following results:

UALB	LALB
$f_1^{1} = 3.57$,where $\left(x_1^{\,F}, x_2^{\,F}, x_3^{\,F}, x_4^{\,F}, x_5^{\,F},\right.$ (1,0,0,0,1.333333,0).	f_1^{2}=6.315,where $\left(x_1^{\,F}, x_2^{\,F}, x_3^{\,F}, x_4^{\,F}, x_5^{\,F}, x\right.$ (2.666667,0,0,0,2.5,0).
UAUB	**LAUB**
f_1^{4}=37.59,where $\left(x_1^{\,F}, x_2^{\,F}, x_3^{\,F}, x_4^{\,F}, x_5^{\,F},\right.$ (20,0,0,0,18,0).	f_1^{3}=21.66,where $\left(x_1^{\,F}, x_2^{\,F}, x_3^{\,F}, x_4^{\,F}, x_5^{\,F}, x\right.$ (7.5,0,0,0,15,0).

Table (3): Results of Applying The Decomposition Algorithm on Linear Functions of The FLDM

SLDM problem using Taylor's series and decomposition algorithm

Now set $x_1^{\,F} = [[2.666667, 7.5],[1,20]]$ and $x_2^{\,F} = [[0,0],[0,0]] = 0$ to the SLDM constraints and get the following results:

UALB	LALB
$f_2^{1} = 1.43033$,where $\left(x_3^{\,S}, x_4^{\,S}, x_5^{\,S}, x_6^{\,S}\right) = ($	f_2^{2}=1.624,where $\left(x_3^{\,S}, x_4^{\,S}, x_5^{\,S}, x_6^{\,S}\right) = (0,0,2.5,0).$
UAUB	**LAUB**
f_2^{4}=4.197,where $\left(x_3^{\,S}, x_4^{\,S}, x_5^{\,S}, x_6^{\,S}\right) = ($	f_2^{3}=3.699,where $\left(x_3^{\,S}, x_4^{\,S}, x_5^{\,S}, x_6^{\,S}\right) = (0,0,15,0).$

Table (3): Results of Applying Taylor's Series and Decomposition Algorithm of the SLDM

TLDM problem using Taylor's series and decomposition algorithm

Now set $x_1^{\,F} = [[2.666667, 7.5],[1,20]]$, $x_2^{\,F} = [[0,0],[0,0]] = 0$ $x_3^{\,S} = [[0,0],[0,0]] = 0$ and $x_4^{\,S} = [[0,0],[0,0]] = 0$ to the TLDM constraints and get the following results:

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UALB	LALB
$f_3^1 = 1.032$,where $\left(x_5^T, x_6^T\right) = (0,1)$.	f_3^2=2.13567,where $\left(x_5^T, x_6^T\right) = (0,1.6666$
UAUB	**LAUB**
f_3^4=26.238,where $\left(x_5^T, x_6^T\right) = (0,18)$.	f_3^3=11.2715,where $\left(x_5^T, x_6^T\right) = (15,0)$.

Table (4): Results of Applying Taylor's Series and Decomposition Algorithm of the TLDM

Conclusion

This paper has presented a three level large scale fractional linear programming problem with rough parameters in constraints. A three level programming problem can be thought as a static version of the Stackelberg strategy. a proposed model for solving the three-level decision model has been presented. At each level we attempted to optimize its problem separately as a large scale programming problem using a decomposition method. Therefore, we handle the optimization process through a series of sub problems that can be solved independently, also the problem of rough parameters has been solved by applying the interval concept to such problem Finally, a numerical example was given to clarify the main results developed in this paper.

However, there are many other aspects, which should by explored and studied in the area of a large scale multi-level optimization such as: 1- nonlinear programming problem with rough parameters in objective and constraints. 2- Large scale multi-level non-linear programming problem with rough parameters in the objective functions and in the constraints.

References

[1] Y. J. Lai, "Hierarchical optimization: a satisfactory solution," Fuzzy Sets and Systems, Vol. 77 No. 3, (1996), pp. 321-335.

[2] M. Osman, M. Abo-Sinna, A. Amer , O. Emam, "A multi-level non-linear multi-objective decision-making under fuzziness", applied mathematics and computation. 153, (2004),pp.239-252.

[3] P. Kundu, S. Kar, M. Maiti, "Some Solid transportation Model with Crisp and Rough Costs", World Academy of Science, Engineering and Technology, vol. 73, (2013),pp. 185-192.

[4] S. Pramanik, and T. K. Roy, "Fuzzy goal programming approach to multi-level programming problems," European Journal of Operational Research, Vol. 176 (2), (2007), pp.1151-1166.

[5] N. Arbaiy, and J. Watada, "Fuzzy goal programming for multi-level multi-objective problem: an additive model," Software Engineering and Computer Systems Communication in Computer and Information Sci-ence, Vol. 180, (2011), pp. 81-95.

[6] O.M. Saad and O.E. Emam, "Taylor Series Approach for Solving Chance-Constrained Bi-Level Integer Linear Fractional Programming Problem", International Journal of Mathematical Archive, 2(5) (2011), 675-680.

[7] A. Hamazehee, M. A. Yaghoobi and M. Mashinchi, "Linear Programming with Rough Interval Coefficients", Journal of Intelligent and Fuzzy Systems, 26(2014), 1179-1189.

[8] H. T. Taha, "Operation Research-An Introduction", 6th Edition, Mac Milan Publishing Co, New York, 1997.

[9] T. I. Sultan, O. E. Emam and A. A. Abohany, "A Decomposition Algorithm for Solving a Three–level Large Scale Linear Programming Problem", Applied Mathematics and Information Science, 5(2014), 2217-2223.

[10] G. Dantzig, and P. Wolfe, The decomposition algorithm for linear programs, Econometrics, 9, 767-778 (1961).

[11] M. Saraj and N. Safaei, Solving bi-level programming problems on using global criterion method with an interval approach, Applied Mathematical Sciences, 6, 1135-1141 (2012).

[12] M. Saraj and N. Safaei1, Fuzzy linear fractional bi-level multi-objective programming problems, International Journal of Applied Mathematical Research, 4, 643-658 (2012).

[13] A. Gaur and S. R. Arora, Multi-level multi-Objective integer linear programming problem, Advanced Modeling and Optimization, 10, 297-322 (2008)

[14] O. Emam, Interactive approach to bi-level integer multi-objective fractional programming problem, Applied Mathematics and Computation, 223, 17-24 (2013).