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A Delaunay triangulation optimization algorithm based on Point-by-point insertion method

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Abstract

In order to solve the problem that mass discrete points to construct Delaunay triangles are slow, a Delaunay triangulation optimization algorithm based on the Point-by-point insertion method is proposed to optimize the fast positioning method of the triangle where the insertion point is located. First, the grid is used to partition the set of points and then inserted into discrete points according to the S-type path so as to reduce the number of triangular effect domain reconstruction, and then empty circumference detection is performed around the insertion point to find a Delaunay characteristics edge, finally, by means of the area coordinate method to locate the edge, the triangle where the insertion point located can be find quickly. The simulation results show that the improved algorithm can balance the performance of time and space, and can deal with a large amount of data stably. In addition, it avoids the existence of different paths in the process of rapid positioning of the triangle, with good applicability.

Introduction

The Delaunay triangulation of the 2D planar domain has the minimum angular maximum and empty circumscribed circle properties, which can maximize the guarantee that the triangles satisfy the approximation of the equilateral (angular) in the Delaunay triangles, there are significantly uniqueness and optimality in the study of discrete data. Delaunay triangulation has been widely used in the fields of scientific computing visualization, computer vision, finite element analysis, route selection, geographical analysis, 3D visualization, geographic information system, virtual reality and map synthesis, and has become the research focus of many scholars[1-5].

The main algorithms of Delaunay triangulation are triangulation growth method, divide and conquer algorithm and point-by-point insertion method[6]. Triangulation growth method is less used because of the lower efficiency, less improved space. Divide and conquer algorithm is recursively divides the set of points until the subset is decomposed into a single triangle, and then the top-down is step merged to generate the final Delaunay triangulation network. The advantage is that it has high efficiency, can quickly and efficiency find the triangular containing the insertion point, and reduce the number of times to determine the LOP; The disadvantages is that due to the use of recursion, the process will take up a lot of memory. The most widely used is the Point-by-point insertion method, which is to create a convex hull containing all data points, and then insert all discrete points in. The algorithm is easier to implement, takes up less memory and is suitable for handling large amounts of data. The procedure for using Point-by-point insert method is as follows:

(1) Record all data points and make rectangular convex hull according to (x_{min},y_{min}), (x_{min},y_{max}), (x_{max},y_{min}), (x_{max},y_{max}), x_{max} and x_{min} are the maximum and minimum values in the horizontal direction, y_{max} and y_{min} are the maximum and minimum values in the vertical direction.
(2) The convex hull area is divided into the appropriate grid size.
(3) Insert the discrete points on the grid in turns;
(4) Locate the triangles where the discrete points are located;
(5) Determine the influence fields of the insertion points and perform the triangle network reconstruction;
(6) Remove the non-existent rectangular convex hull vertices after inserting all discrete points.

For the Point-by-point insertion method, the key step is to quickly find the triangle when it is inserted into the discrete point. The conventional algorithm uses the method of traversing all triangles to determine the position of the insertion point, the time complexity is large, and the running time of the massive data will be multiplied. In recent years, many scholars have proposed different optimization algorithms for this step. The literature[7] proposes a direction search technique, which determines the search direction or terminates the search by judging whether the center of gravity of the triangle and the insertion point are located on the same side or opposite side of a side. However, it is necessary to calculate the center of gravity multiple times, the calculation is complicated and the efficiency is low. Besides, in the case of using the direction search technology, there will be different paths. For this situation, a restriction condition is added to solve this problem in the literature[8], and the insertion point is positioned by judging whether the connection of the center of gravity with the insertion point intersects the triangle of the triangle, but the calculation of whether the intersection increases the amount of calculation. Besides, in the case of using the direction search technology, there will be different

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paths. For this situation, a restriction condition is added to solve this problem in the literature[8]. The insertion point is positioned by judging whether the connection line between the center of gravity and the insertion point intersects the triangle of the edge, but the amount of computation is increased. The topological relation between the point and the triangle is used to reduce the number of calculations for the center of gravity point line relation localization algorithm in the literature[9], but ignore the consideration of different paths. Based on the direction search technique, the new generated triangle is used as the initial triangles in the literature[7], thus shortening the positioning process of the search path, but the positioning efficiency is not high because the initial triangle is uncertain. Insert into the known point in ascending order of coordinates in literature[11], because of its insertion point only in one side of the rectangle vertex, it reduces the judgment number of positional relationship and also to avoid the positioning of the triangle, but increased the number of LOP processing. In literature[12], the convex hull is constructed by using the triangular growth method, and the initial triangulation is generated by the generated temporary edge. Data segmentation is clever, but the algorithm requires a lot of LOP processing, which leads to the overall efficiency is not high. In this paper, the characteristics of the Delaunay triangulation, the area coordinate method and the topological relation between the point and the triangle are used to locate the triangle where the insertion point is located, and the path with different positioning is avoided.

This reduces the time of positioning the triangle where the insertion point is located and improves the efficiency of the Point-by-point insertion method.

Improvements to Point-by-point insertion

A. Use the grid to block the set of points

The convex hull is divided and processed into rectangular grids. Each insertion point is first positioned to the grid and then searched from the grid so as to quickly reduce the scope of the insertion point, saving time for fast positioning. The convex hull division takes into account the following three points: 1. The grid contains all triangles and discrete data; 2. The grid can not be too small, otherwise it will lead to some grid without triangles, increase the number of queries; 3. Grids can not be too large, otherwise it will cause some of the triangles in the grid too much, affecting query efficiency. Set the total number of discrete points to n, and the grid threshold is k = 50, Use the following formula:

$$s = \sqrt{\frac{(x_{\max} - x_{min})(y_{max} - y_{min}) * k}{n}} \tag{1}$$

$$a = (int)\frac{x_{max} - x_{min}}{s} \tag{2}$$

$$b = (int)\frac{y_{max} - y_{min}}{s} \tag{3}$$

S represents the size of the grid; a, b represents the number of meshes in the direction of the vertical and horizontal coordinates.

B. Insert discrete points into the grid in turns

The Delaunay triangulation is uniquely determined by a given set of points, so the order of addition does not affect the final triangular mesh shape, but the different order of addition will affect the Delaunay triangular grid generation time. The literature[13]proves that in the case of the Delaunay triangulation generation method unchanged, the search time can be reduced if the interpolation point is inserted according to the adjacent principle. However, after determining the triangular position of the insertion point, the number of triangular domain reconstructions will be greatly increased. If the insertion points are not sorted by the adjacent principle, but by the uniform distribution of the sorting principle, the number of domain reconstructions can be significantly reduced, but the point search time is longer. In this paper, convex hulls is divided into grids and then insert the discrete points into the grid in turns. This not only takes into account the principle of uniform distribution, but also makes the distance between the insertion point is not too far. Considering the association between the new insertion point and the previous insertion point, the S-type path is used for the insertion point.

C. Quickly position the triangle where the insertion point is located

An important part of the point-by-point insertion method is to quickly position the triangle that contains the insertion point. The usual method is to randomly find a triangle as the initial triangle, the use of the topological relationship between the point and the triangle, and then by the triangular center of gravity or triangle area coordinates to determine the insertion point from the triangle position relationship, start from the initial triangle, follow the direction of the triangle where the insertion point is located, search for it, and finally find the triangle. During the positioning process,

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there will be different paths if the insertion point is between the regions of the relative extension lines on either side of the triangle.

The algorithm skips the search for triangles, searching for a Delaunay edge arbitrarily in the grid to which the insertion point belongs. If the edge can form a Delaunay triangle with the insertion point, then the edge is the request edge. If the combination of triangles does not conform characteristics that any four points do not in a circle , the edge is not the edge of the request, and then start from the insertion point, along the direction from the insertion point to the endpoint of edge, the first Delaunay side of the search is the desired edge. There are only two cases after finding the edge: If the edge of the triangle only one, then directly locate the triangle; If there are two triangles, the common edge of two triangles are judged separately by insertion points using area coordinate method. The triangle with the result greater than zero is the triangle where the insertion point is located. The special case is that when the insertion point happens to be at the intersection of the Delaunay edge or the Delaunay edge, any triangle containing the Delaunay edge is eligible.

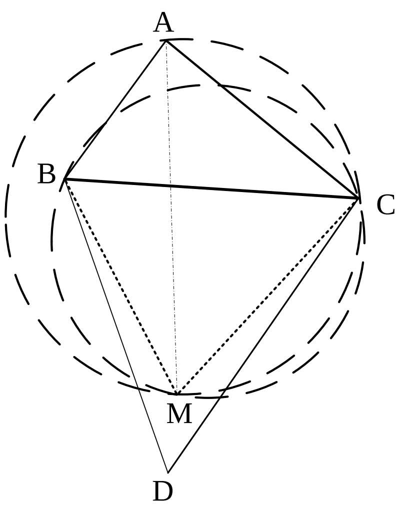

Figure 1. Quickly position the triangle

As shown in figure 1: The original part of the Delaunay triangulation is ABCD, the insertion point is M. To find the existence of Delaunay edge of the grid, the first to take the AC edge of the test, verified, △ABC's circumscribed circle which also contains the B point, does not meet the empty round characteristics, so this edge is not a request edge. Then starting from point M, the first Delaunay edge is the BC edge found along the direction of the MA, and the triangle with BC edge is determined by using area coordinate method. It can be found that △BCD is the triangle where the insertion point M is located.

a. Empty circumference detection

Whether the triangle is a Delaunay triangle can be determined by verifying the empty circle. Likewise, the nature can be understood as follows: If there is a circle: it passes through the two ends of the Delaunay side near the insertion point and the insertion point, and there are no other points inside the circle, these three points can form a triangle that conforms to the Delaunay property.

Common methods are to calculate the radius of the triangle and its external round center, and then through the comparison point to the center of the distance and the radius of the circumscribed circle so as to determine the relationship between the point and the circumcircle. The process involves many operations, such as extraction of root, square and trigonometric function, which will affect the program's time efficiency. As shown in figure 2, based on the diagonal complementary properties of circular quadrangle, it can be known that $\alpha + \beta = 180^\circ$, that is, $cos\alpha + cos\,\beta = 0$. The cosine value is then calculated by the vector dot product formula. Based on the above theory, if $cos\alpha + cos\gamma > 0$, point E is outside of the circle; if $cos\alpha + cos\,\gamma = 0$, point E is on the circle; if $cos\,\alpha + cos\,\gamma < 0$, point E is in the circle.

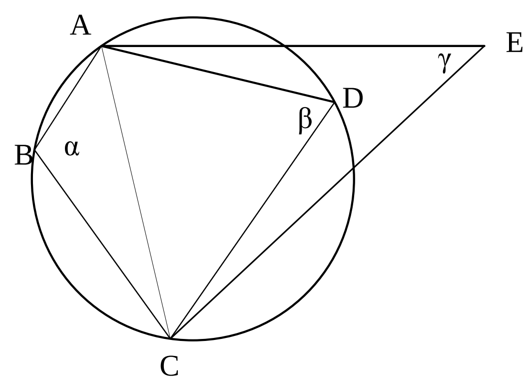

Figure 2. Triangle area coordinates

b.Area coordinate method

As the figure 3 shows, assume that the coordinates of △ABC are A(x1, y1), B(x2, y2), C(x3, y3), P(x, y)is any point within the triangle, △PBC, △PBC, △PCA area ratio: k1: k2: k3, and let k1+k2+k3=1, Then the area coordinates of point P are (k1, k2, k3). Area coordinates are a linear transformation, within the triangle, all coordinates belong to the open interval (0, 1); At the edge of the triangle, at least one coordinate is 0 and the rest is in the closed interval [0, 1]; If the triangle is outside the corresponding edge, the coordi-

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nates are less than 0. When the △ABC vertices are arranged in a counterclockwise direction, the area coordinates can be reduced to:

$$p1 = (y3 - y2)(x - x2) - (y - y2)(x3 - x2) \quad (4)$$
$$p2 = (y1 - y3)(x - x3) - (y - y3)(x1 - x3) \quad (5)$$
$$p3 = (y2 - y1)(x - x1) - (y - y1)(x2 - x1) \quad (6)$$

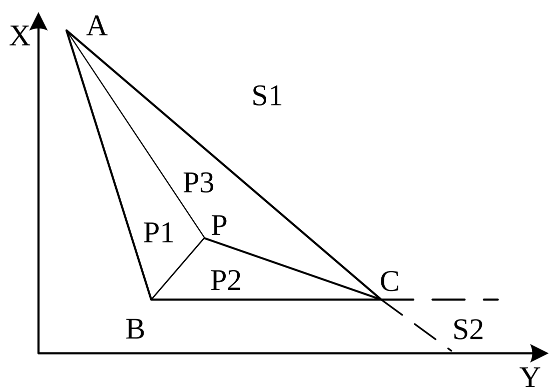

Figure 3. Area coordinate method

The positional relationship between the insertion point and one side of the triangle can be determined by the sign of the area coordinates. If any point P is in the triangle ABC, Its area coordinates is: P1>0, P2>0, P3>0; If P is in area S1, Its area coordinates is: P1>0, P2<0, P3>0: If P is in area S2, Its area coordinates is: P1<0, P2<0, P3>0.

Results and Analysis

This paper uses *C++* as the implementation language of the algorithm, the experimental environment for this experiment is a personal desktop, its configuration is as follows: *CPU: Intel i3-2130 3. 40G*, *RAM: 4. 00GB*, operating system: win10, In order to test the efficiency and stability of the algorithm, two experiments were designed to analyze the running results.

Experiment one: By generating the time test for the points of different orders of magnitude, the relationship between the number of points and the generation time is obtained as the figure4 shows.

It can be seen from the figure that when the number of points increases, the efficiency of the algorithm is still very high, in dealing with massive data, the algorithm advantage is more obvious.This also proves that the algorithm is adapted to the construction of a large number of sets of Delaunay triangles.

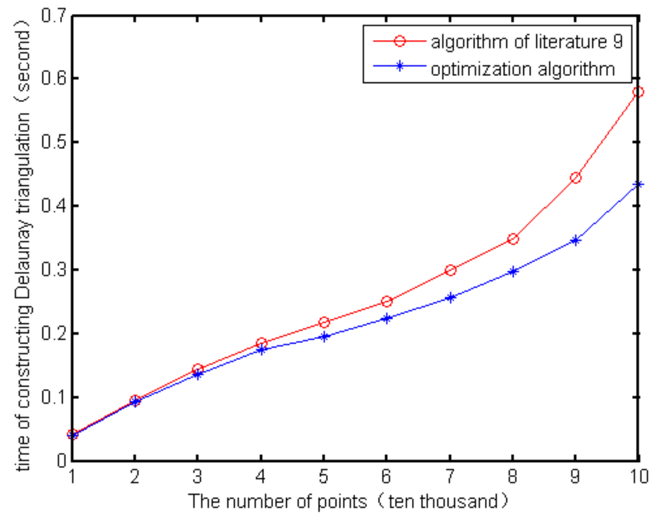

Figure 4. Experiment one

Experiment two: The algorithm is used to construct the Delaunay triangulation, and the difference is averaged at 50, 000 points, 100, 000 points, and 200, 000 points. By comparing the different generation times in order to test the robustness of the algorithm.

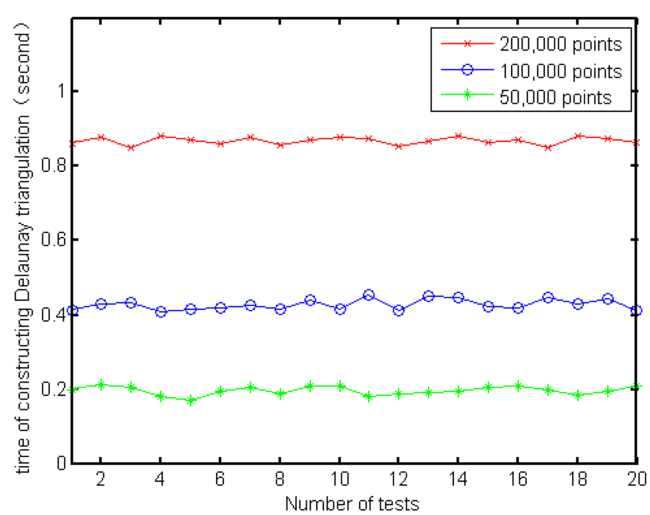

Figure 5. Experiment two

As shown in figure 5, by constructing the large number of points set of the same point, it can find that the generation time fluctuation is small and basically maintained in a certain range.This also proves that the algorithm has good robustness and is suitable for large amounts of data.

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Conclusions

In this paper, a Delaunay triangulation optimization algorithm based on Point-by-point interpolation is proposed, which provides a new search method for the fast positioning of the insertion point, which improves the positioning speed of the triangle where the insertion point is located. By analyzing the efficiency of the algorithm and constructing the Delaunay triangular time, it shows that the optimization algorithm is robust and adaptable, and achieves the better level of Delaunay triangulation.

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Biographies

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