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ADE-TLM SIMULATION OF WAVE PROPAGATION IN NONLINEAR GAIN MEDIA

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Abstract

This paper presents a computational model of electrons dynamics and gain in four-level atomic systems. This approach is based on the existing Auxiliary Differential Equation (ADE) technique in the context of the Transmission Line Matrix method with the symmetrical condensed node (SCN -TLM) and novel voltage sources. The scattering matrix characterizing the SCN with the new voltage sources is provided and the numerical results are compared with those of the literature or with the theoretical ones.

Introduction

During the propagation of an electromagnetic (EM) wave in a medium, it can be submitted to three processes: spontaneous emission, stimulated absorption and emission. These processes contribute, according to the rate equations, to the evolution of the population densities of electron energy states during propagation [1]. On the other hand, the space-time evolution of EM waves, during propagation, obeys Maxwell's equations, and depends on the macroscopic polarization density owing to the response of the medium to the EM wave. When the field has a low intensity, the polarization density is supposed proportional to the electric field intensity. This kind of non-linear interaction allows the exchange of energy between EM waves and the electrons of the medium. Several papers have modeled these interactions using FDTD method [2]-[5], in particular the approach based on the auxiliary differential equation (ADE), to study absorption in two-energy level atomic systems and gain of four-level systems [2]. By coupling Maxwell's equations and the rate equations describing electrons population, this method can model EM wave propagation in random gain media [3]. It was also used, by including Pauli Exclusion Principle and the dynamic pumping, to study EM wave interaction with four-level two-electron atomic systems [4] and multi-level multi-electron atomic systems [5].

A novel algorithm based on the Transmission Line Matrix (TLM) method [6]-[11] with condensed symmetrical node (SCN) and new voltage sources combined with the auxiliary differential equation (ADE) was rarely used for modeling this kind of phenomena. The first approach modeling quantum properties of two energy level atomic systems were proposed in [8]. The proposed ADE-TLM model describes the space-time evolution of electrons populations densities in each energy level and gives the frequency evolution of the gain in four-level atomic system. Furthermore, the scattering matrix characterizing the SCN with the new voltage sources is provided and the simulation's results are compared to those of the literature or obtained by the theoretical solutions.

Theory

In our proposed model, we reformulate the auxiliary differential equation (ADE) technique in the context of the TLM method. This approach allows the simulation of the propagation of optical pulses in four-level atomic systems, by solving Maxwell's and polarization equations describing the effects of the media on the EM wave's propagation. This approach is based on the time discretization applied to the components of the EM field and polarization vector by using centered differencing.

A. Macroscopic polarization equations

Electron transitions in atomic systems with four energy levels E_i and respective normalized densities $N_i (i = 0, 1, 2, 3)$ are considered as in a simplified model of two oscillating dipoles: P_a for E_1 to E_2 transitions with a frequency ω_a and P_b for E_0 to E_3 transitions with a

pumping frequency ω_b illustrated in Figure 1. (2)

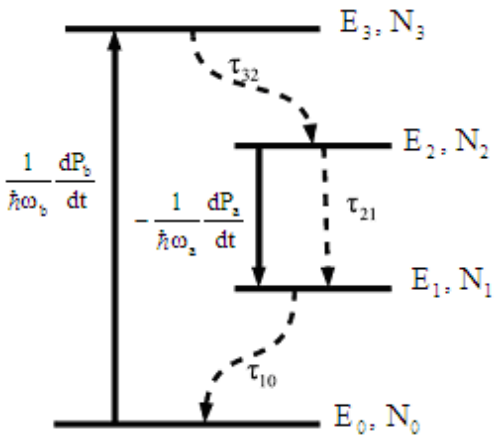

Figure 1.Four-level atomic systems model.

The equations of macroscopic polarization are governed by the following auxiliary differential equation: [4]:

$$\frac{d^2\overline{P_a}(t)}{dt^2} + \Delta\omega_a \frac{d\overline{P_a}(t)}{dt} + \omega_a{}^2 \overline{P_a}(t) = k_a(N_1 - N_2)\overline{E}(t)$$
$$\frac{d^2\overline{P_b}(t)}{dt^2} + \Delta\omega_b \frac{d\overline{P_b}(t)}{dt} + \omega_b{}^2 \overline{P_b}(t) = k_b(N_0 - N_3)\overline{E}(t) \quad (2)$$

Where $k_a = \frac{6\pi\varepsilon_0 c^3}{\omega_a{}^2}\gamma_{ra}$, $k_b = \frac{6\pi\varepsilon_0 c^3}{\omega_b{}^2}\gamma_{rb}$

ε_0 is the permittivity of free space and $\gamma_{ra} = \frac{1}{\tau_{21}}$, $\gamma_{rb} = \frac{1}{\tau_{30}}$ are the radiative energy decay rates.

$\Delta\omega_a$ and $\Delta\omega_b$ are the total energy attenuation rate which describe the spectral line width of the transition taking into account the energy loss by pumping and relaxation effects.

In four-level atomic systems $E_0 < E_1 < E_2 < E_3$ and population densities N_0, N_1, N_2 and N_3 N_3 respectively (depicted in Figure 1), the rate equations are [2]:

$$\frac{dN_3}{dt} = -\frac{N_3}{\tau_{30}} + \frac{1}{\hbar\omega_b}\overline{E}\frac{d\overline{P_b}}{dt}$$
$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} + \frac{1}{\hbar\omega_a}\overline{E}\frac{d\overline{P_a}}{dt}$$
$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_{10}} - \frac{1}{\hbar\omega_a}\overline{E}\frac{d\overline{P_a}}{dt}$$
$$\frac{dN_0}{dt} = \frac{N_3}{\tau_{30}} - \frac{1}{\hbar\omega_b}\overline{E}\frac{d\overline{P_b}}{dt}$$

B. ADE-TLM algorithm

In order to implement a numerical model for the simulation of the interaction of EM wave with four-energy level atomic systems using the TLM method with the SCN and novel voltage sources, we combine the equations (1) and (2) with those of Maxwell:

$$\nabla\Lambda\overline{H} = \varepsilon_0\frac{\partial\overline{E}(t)}{\partial t} + \frac{\partial(\overline{P_a}(t)+\overline{P_b}(t))}{\partial t}$$
$$\nabla\Lambda\overline{E} = -\mu_0\frac{\partial\overline{H}(t)}{\partial t} \quad (3)$$

Let's consider an EM wave propagating in a 3D regular mesh $(\Delta x = \Delta y = \Delta z)$, Maxwell's equations time-discretization with a time step Δt gives at a time $t^n = n\Delta t$, the following expressions:

$$E^{n+1}(i,j,k) = E^n(i,j,k) - \frac{1}{\varepsilon_0}(P_t^{n+1}(i,j,k) - P_t^n(i,j,k)) + \frac{\Delta t}{\varepsilon_0}\nabla\Lambda H^{n+\frac{1}{2}}(i,j,k)$$
$$H^{n+1}(i,j,k) = H^n(i,j,k) - \frac{\Delta t}{\mu_0}\nabla\Lambda E^{n+\frac{1}{2}}(i,j,k) \quad (4)$$

Where $P_t(i,j,k) = P_t(i,j,k) + P_t(i,j,k)$.

In these equations, we have replaced the EM parameters (E, H) by their equivalent voltages and currents (V, I):

$$V = E.\Delta L,\ I = H.\ \Delta L \text{ and } V = Z_0.I \quad (5)$$

Where Δl is the space step and Z_0 is the intrinsic impedance of vacuum. Thus, the equations (4) become:

$$V^{n+1}(i,j,k) = V^n(i,j,k) - \frac{\Delta l}{\varepsilon_0}(P_t^{n+1}(i,j,k) - P_t^n(i,j,k)) + \frac{\Delta t}{\varepsilon_0}\Delta l.\nabla\Lambda H^{n+\frac{1}{2}}(i,j,k)$$
$$I^{n+1}(i,j,k) = I^n(i,j,k) - \frac{\Delta t}{\mu_0}\Delta l.\nabla\Lambda E^{n+\frac{1}{2}}(i,j,k) \quad (6)$$

The fields (E, H) in the curls of equations (6) are converted into local incident and scattered voltage pulses V^i and V^r on the faces of the SCN of the TLM Method [6]-[7]. Thus making use of charge and energy conservation principle through the transmission lines forming the SCN, and imposing the continuity conditions on the electric and magnetic

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fields [6], we obtain at time $n\Delta t$ the following scattering matrix of the SCN with voltage sources:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \end{pmatrix}^r = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \end{pmatrix}^i + \frac{1}{4} \begin{pmatrix} V_{sx} & 0 & 0 \\ V_{sx} & 0 & 0 \\ 0 & V_{sy} & 0 \\ 0 & V_{sy} & 0 \\ 0 & 0 & V_{sz} \\ 0 & 0 & V_{sz} \\ 0 & 0 & V_{sz} \\ 0 & V_{sy} & 0 \\ V_{sx} & 0 & 0 \\ 0 & 0 & V_{sz} \\ 0 & V_{sy} & 0 \\ V_{sx} & 0 & 0 \end{pmatrix} \quad (7)$$

The obtained matrix models EM wave propagation in an atomic system taking into account the physical effects related with the medium polarization with the voltage sources (V_{sx}, V_{sy}, V_{sz}) which are expressed as follows:

$$V_{su}^{n+1} + V_{su}^{n} = -\frac{\Delta l}{\varepsilon_0}\left(P_{tu}^{n+1}(i,j,k) - P_{tu}^{n}(i,j,k)\right) \quad (8)$$

Where u=x, y, z. P^{n+1} is the medium's polarization at time $t^{n+1} = (n+1)\Delta t$. Its expressions are deduced from the time discretization of equations (1):

$$P_{au}^{n+1}(i,j,k) = \frac{2\Delta t^2}{2+\Delta\omega_a \Delta t}\left[\begin{array}{l} \left(\frac{2}{\Delta t^2} - \omega_a^{\,2}\right)P_{au}^{n}(i,j,k) \\ + \left(\frac{\Delta\omega_a}{2\Delta t} - \frac{1}{\Delta t^2}\right)P_{au}^{n-1}(i,j,k) \\ + k_a\left(N_{1u}^{n} - N_{2u}^{n}\right)\frac{V_u^{n}(i,j,k)}{\Delta l} \end{array}\right]$$

$$P_{bu}^{n+1}(i,j,k) = \frac{2\Delta t^2}{2+\Delta\omega_b \Delta t}\left[\begin{array}{l} \left(\frac{2}{\Delta t^2} - \omega_b^{\,2}\right)P_{bu}^{n}(i,j,k) \\ + \left(\frac{\Delta\omega_b}{2\Delta t} - \frac{1}{\Delta t^2}\right)P_{bu}^{n-1}(i,j,k) \\ + k_b\left(N_{0u}^{n} - N_{3u}^{n}\right)\frac{V_u^{n}(i,j,k)}{\Delta l} \end{array}\right] \quad (9)$$

The TLM method with voltage sources algorithm implementation is based on a recursive computation of $P_{au}^{n+1}(i, j, k)$ and $P_{bu}^{n+1}(i, j, k)$ given by the equation (9). This allows to update voltage sources given by (8), $V^{n+1}(i, j, k)$ from (6) and to determine the population densities N_i^{n+1} with i = 0, 1, 2, 3 given by (2). Local scattered voltage pulses V^r at the SCN are obtained from the scattering matrix expressed in (7). Finally, we establish connections between nodes along the spatial TLM lattice.

Numerical results

The proposed model was used to simulate the effect of an incident EM wave on a four-level atomic system.. The physical parameters characterizing this atomic system of four energy levels $E_0 < E_1 < E_2 < E_3$ are [2]:

$$\omega_a = 2\pi.10^{14}\ rad/s\ ,\ \omega_b = 2\pi.10^{16}\ rad/s\ ,$$

$$\Delta\omega_a = \Delta\omega_b = 4\pi.10^{12}\ rad/s\ ,$$

With the electron life-time between energy levels $\tau_{32} = 0,99.10^{-10}\ s$, $\tau_{21} = 1,35.10^{-7}\ s$, $\tau_{10} = 1.10^{-9}\ s$, $\tau_{30} = 1.10^{-10}$ s, and initial normalized electrons population densities in the energy level $N_0 = N_1 = 1$ and $N_2 = 0$. The spatial TLM lattice considered is $(1,1,1000)\Delta l$, with Δl is the mesh width taken to be $3.10^{-2}\ \mu m$. The atomic system spans the cells located between $8\Delta l$ and $208\Delta l$ in the z direction. The air-system interface is excited first with a sine wave of amplitude 2.10^8V/m and frequency 1.10^{14} Hz. In order to prove the efficiency of the proposed model, we present in Figure 2 the time evolution of the population difference $\Delta N_{12} = N_1 - N_2$ at $z = 108\Delta l$. The results obtained from TLM method show good agreement with FDTD method [2].

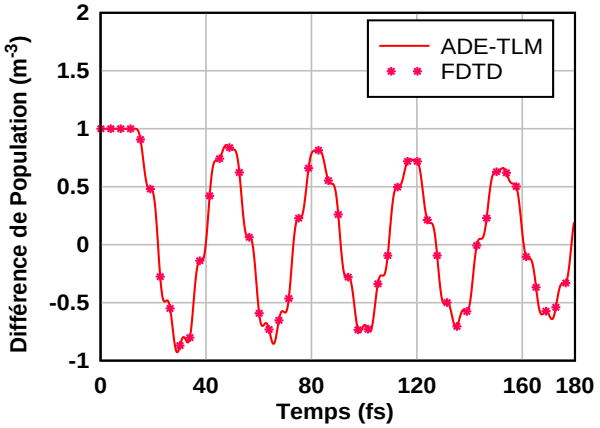

Figure 2. Comparison of population difference as computedted by ADE-TLM and FDTD methods.

In the second part, we have excited the air-system interface using a Gaussian wave of amplitude 1V/m and frequency 1.10^{14} Hz [2]. The values of the electric field magnitudes in the two edges, air-system and system-air, allow to evaluate the amplification factor as shown in Figure 3. Here again, the TLM method generates results in very good agreement with those of [2] and with the theoretical ones [1].

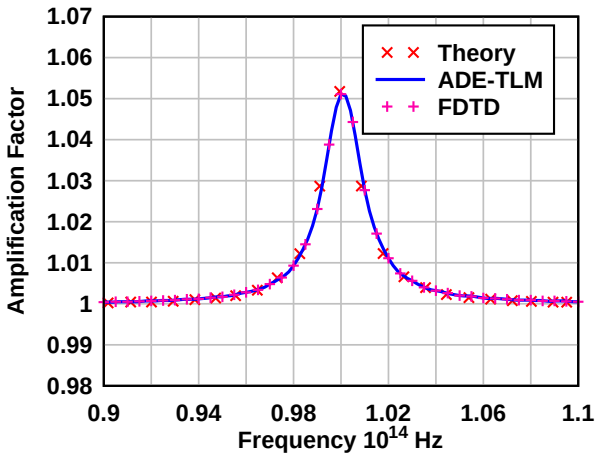

Figure 3. Comparison of amplification factor versus frequency as computed by theory, FDTD and ADE-TLM methods.

Conclusion

We have developed a novel modeling approach for EM wave propagation in four-level atomic systems, using the ADE-TLM method, with new voltage sources. In order to illustrate the validity of this novel model combining both classical electrodynamics with quantum mechanics, we have simulated the electrons dynamic between four energy levels through the variation of populations' densities by the rate equations. The good agreement between the novel TLM approach results and those available in the literature proves its validity and efficiency. Hence, this study contributes to the widening of the field of the problems treated by the TLM method.

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